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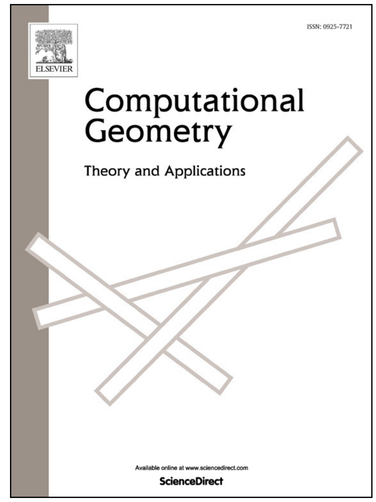

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A LINEARIZED CIRCLE PACKING ALGORITHM

CHARLES COLLINS, GERALD L. ORICK, AND KENNETH STEPHENSON

Abstract. This paper presents a geometric algorithm for approximating radii and centers for a variety of univalent circle packings, including maximal circle packings on the unit disc and the sphere and certain polygonal circle packings in the plane. This method involves an iterative process which alternates between estimates of circle radii and locations of circle centers. The algorithm employs sparse linear systems and in practice achieves a consistent linear convergence rate that is far superior to traditional packing methods. It is deployed in a MATLAB® package which is freely available. This paper gives background on circle packing, a description of the linearized algorithm, illustrations of its use, sample performance data, and remaining challenges.

A *circle packing* is a configuration of circles with a specified pattern of tangencies; in our case a triangulation of a topological disc or sphere [34]. It is important to distinguish its *combinatorial* and *geometric* structures. The pattern of tangencies is given as an abstract simplicial 2-complex K, a combinatoric object with no inherent geometry, whereas a circle packing P is a concrete configuration of circles — specifically, circles tangent in the pattern of K. Our packings will be *univalent*, meaning that the circles have mutually disjoint interiors. By connecting centers of tangent circles with geodesic segments, P provides an embedding of K. It is in this sense that a univalent circle packing P imposes a geometric structure on a combinatorial object K.

A toy example is illustrated in Figure 1: there is a hand sketch of the abstract complex K, a generic univalent packing P, and the maximal packing P_K in the unit disc $\mathbb{D}$, one of the three types of circle packings targeted by the new algorithm. Though traditional methods easily handle simple cases like this, the linearized algorithm, originating with the second author [20] and known as `GOpack`, is aimed at problems of much greater size and complexity. Note in Figure 1 that the geometric triangulations (shaded) formed by the packings are embeddings of K.

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