Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Optimal experimental design on the loading frequency for a probabilistic fatigue model for plain and fibre-reinforced concrete

M.J. Rivas-López [a], R.C. Yu [b,*], J. López-Fidalgo [c], G. Ruiz [b]

[a] Departamento de Estadística, Universidad de Salamanca, Plaza de los Caídos s/n, 37008 Salamanca, Spain
[b] ETS de Ingenieros de Caminos, C. y P., Universidad de Castilla-La Mancha, Avenida Camilo José Cela s/n, 13071 Ciudad Real, Spain
[c] Departamento de Matemáticas, Universidad de Castilla-La Mancha, Avenida Camilo José Cela s/n, 13071 Ciudad Real, Spain

HIGHLIGHTS

- Develop a general procedure for optimal design of fatigue characterisation.
- Derive FIM for chosen fatigue models to improve design efficiency.
- Perform robustness analysis to evaluate design efficiency.

ARTICLE INFO

Article history:
Received 6 March 2016
Received in revised form 18 August 2016
Accepted 19 August 2016
Available online xxxx

Keywords:
Fisher Information Matrix (FIM)
D-optimality
Weibull cumulative distribution function
Loading frequency
Fatigue model

ABSTRACT

The objective is to improve the fatigue characterisation process based on the concept of optimal experimental design. This is carried out through a probabilistic model, previously developed, which takes into account the experimentally observed loading frequency effect on the fatigue life in plain and fibre-reinforced concrete. The Fisher Information Matrix is first obtained for the simplified fatigue model. The optimal design is found to be located at the minimum values allowed for both the maximum stress and stress ratio, whereas the two loading frequencies are the minimum and maximum values in the defined range. Next, the FIM is derived for the extended fatigue model. The previously carried out experimental plan is 65% efficient compared to the optimum. Even though it has been developed for the specific chosen fatigue model, the current procedure can be applied to any other fatigue model to significantly improve the fatigue characterisation process of any material.

1. Introduction

Fatigue tests are known to be time consuming, and can sometimes be unachievable if not properly designed. The procedure on how to determine the number of tests needed to characterise certain materials is an open issue. In the current work, we make use of the Fisher Information Matrix (FIM) to derive the optimal location of tests to characterise fatigue performance of concrete-related materials under given loading conditions. In particular, the fatigue model based on an initial distribution developed by Saucedo et al. (2013) is chosen as an example to carry out the optimal design process. The developed methodology, however, can be applied to any other given fatigue model.

* Corresponding author.
E-mail address: rena@uclm.es (R.C. Yu).

http://dx.doi.org/10.1016/j.csda.2016.08.014
0167-9473/

Development of high-performance concrete for structures undergoing dynamic and cyclic loading has led to experiments conducted to study the influence of different fatigue parameters (Graf and Brenner, 1934; Kesler, 1953; Nordby, 1958; Rusch, 1960; Mudock, 1965; Aas-Jakobsen, 1970; Awad and Hilsdorf, 1971; Sparks and Menzies, 1973; ACI Committee, 1974; Tepfers and Kutti, 1979; Tepfers, 1979; Hsu, 1981; Furtak, 1984; Oh, 1991; Zhang et al., 1996; Li et al., 2007; Zhao et al., 2007; Medeiros et al., 2015). These parameters are related to either the fatigue test conditions, such as the minimum stress, $\sigma_{\min}$, the maximum stress, $\sigma_{\max}$, the loading frequency, f, or the material properties; for example the static material strength, σ_c, (compressive or tensile). In the case of concrete, it has been detected that the influence of the stress ratio, R ($\sigma_{\min}$ divided by $\sigma_{\max}$), the loading frequency, f, and the stress level, $\sigma_{\max}/\sigma_c$, is quite relevant (Aas-Jakobsen, 1970; Oh, 1991; Plekhov et al., 2011; Medeiros et al., 2015).

Although the influence of loading frequency has been observed as early as the 1950s (Kesler, 1953), the loading time (Hsu, 1981) or frequency (Furtak, 1984) was not included in the fatigue equation until the 1980s. Moreover, the static strength in concrete exhibits large dispersion and a Weibull distribution has been considered as the best distribution to fit fatigue life in concrete (Oh, 1991; Li et al., 2007). Recently Saucedo et al. (2013) proposed a fatigue model to take into account the fact that, on the one hand, the static strength of concrete is a Weibull distribution; on the other hand, such a distribution can be considered as the limit behaviour when the fatigue life approaches one cycle. Meanwhile, influences of the loading frequency and of the stress ratio have also been incorporated. The application range of the proposed fatigue model is below 10 Hz according to experimental tests by Ruiz et al. (2011), Medeiros et al. (2015). Depending on the effect of the loading frequency on the hardening exponent of the dynamic material strength is ignored or not, correspondingly the fatigue model is viewed as *simplified* or *complete*. We aim to explore the procedure to calibrate the model parameters based on the concept of optimal experimental design herein.

Optimal experimental designs are especially useful when experimentation is expensive, time consuming or difficult to carry out. A good design will definitely save time, money and provide a better fitting of the model. An experimental design means selecting experimental conditions in such a way that they result in precise estimates or good predictions with a minimum of sample size, which in other words means adequately choosing the levels of the covariates. The optimality of a design is model-oriented in the sense that a design could be quite good for a particular model and not so good for a rival model. In the case of the existence of rival models a discrimination test may be performed to detect the most appropriate one. Optimal designs can be computed for discrimination purposes (López-Fidalgo et al., 2007). In reliability and survival analysis, the Cox proportional hazards model (Cox, 1972) is quite traditional, although proportionality is not always justified. The model is frequently fitted using partial likelihood. Optimal experimental designs in this context were recently computed by López-Fidalgo and Rivas-López (2014). Even though accelerated failure testing models have been increasingly used in recent years, experimental designs have not been studied as much in this field; see Rivas-López et al. (2014). In the current work, optimal experimental design is studied for the aforementioned fatigue model for concrete-like materials; in particular, to best estimate the model parameters under given loading conditions.

The rest of this paper is organised as follows. The concept of optimal experimental design is presented next. Applications to the simplified and complete fatigue model are developed in Sections 3 and 4, respectively. Relevant conclusions are drawn in Section 5.

2. Optimal experimental design

Let x be the vector of covariates, say an experimental condition (loading frequency, maximum stress and stress ratio for a fatigue test), which can be chosen from a compact *design space*, χ; typically a product of intervals. For a value of x, a response variable time-to-event, t, (equivalently, the fatigue life, N), is observed. This is considered as a random variable from a parametric family of distributions, indexed by β, a vector of parameters. An *exact design* of size n is defined by a collection of experimental conditions $x_1, x_2, \ldots, x_n$ in χ, where some of these may be repeated. Thus, a probability measure can be defined with support on the distinct points of the design with weights proportional to the number of repetitions (replicates). This leads to an extension of this definition to any probability measure, ξ, the so called *approximate design* (Kiefer, 1974), which will be used in this work. An exact design of a particular size, say n, is what one can only put in practice using just n experiments. Extending the concept to any probability measure (approximate design) means a kind of abstraction of the real world. This is made because the concept allows a mathematical result, which proves very useful to compute optimal designs. As a matter of fact, computing optimal exact designs is rather difficult and frequently convergence of the typical algorithms cannot be proved. The main drawback is that once the optimal approximate design is computed, an exact design has to be obtained using some rounding procedure. For instance, for a discrete measure, ξ, the experimenter has to perform "approximately" $n_i \approx n\xi(x_i)$ experiments at x_i, in such a way $\sum_i n_i = n$. Imhof et al. (2001) provided some examples to show that if the sample size is large, any rounding procedure leads to a quite efficient exact design. Otherwise, if n is small, then the impact of the rounding may be quite important. Hereafter n will be the total number of experiments to be performed, while k will be the distinct experimental conditions, some of them replicated. An approximate design with k different points in its support will be frequently called a k-point design.

The Fisher information is a way of measuring the amount of information that an observable random variable x contains about an unknown parameter upon which the probability model of x depends. The Fisher Information Matrix (FIM)