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A consistency model for group decision making problems with interval multiplicative preference relations

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ABSTRACT

The main aim of this paper is to present a consistency model for interval multiplicative preference relation (IMPR). To measure the consistency level for IMPR, a referenced consistent IMPR of a given IMPR is defined, which has the minimum logarithmic distance from the given IMPR. Based on the referenced consistent IMPR, the consistency level of an IMPR can be measured and an IMPR with unacceptable consistency can be adjusted by a proposed algorithm such that the revised IMPR is of acceptable consistency. A consistency model for group decision making (GDM) problems with IMPRs is proposed to obtain the collective IMPR with highest consistency level. Numerical examples are provided to illustrate the validity of the proposed approaches in decision making.

1. Introduction

A group decision making (GDM) can be interpreted as obtaining a common solution from a panel of experts or decision makers over a finite set of alternatives. In the GDM process, the decision makers may provide opinions over a set of alternatives by means of various preference relations [2,4,5,7,10,11,13–18,25,27,32,34,35,41,42,44,49,52]. The multiplicative preference relation, originally introduced by Saaty [28], is the most widely used preference relation taking the form of a reciprocal judgment matrix. For constructing such judgment matrix, the decision maker should give a ratio of the preference intensity of one alternative over another when comparing each pair of alternatives. Due to uncertainty and vague, time pressure or lack of expertise knowledge, decision makers sometimes may have difficulty in expressing the ratios as crisp numbers. Alternatively, decision makers think they can better or fully provide their opinion by means of interval values instead of crisp numbers. In such cases, IMPR is one useful tool to express decision makers' preferences.

Many researchers paid their attention to decision making problems with IMPRs. For example, Liu et al. [22] introduced an incomplete IMPR and gave the definitions of consistent and acceptable incomplete ones. In addition, they proposed a goal programming model to complement the acceptable incomplete one. Xu and Liu [43] presented a consensus model for group decision making with IMPRs by using the relative projections of individual preference relations on the collective one. Xu and Cai [46] gave a combined weighting method to derive the weights of decision makers and established linear programming models to derive the weight intervals of alternatives. Wang et al. [38] proposed the compatibility of IMPRs in GDM and applied it to determine the weights of experts. Wang and Elhag [36] proposed a goal programming method to obtain interval weights from an interval comparison matrix, which are assumed to be normalized and can be derived from a goal programming model at a time. Zhang et al. [51] proposed a method to derive the interval priority weights for interval pairwise comparison matrix by extracting consistent matrices with linear or nonlinear programming models. Liu et al. [24] gave a transformation formula between interval fuzzy and multiplicative preference relations and presented an algorithm for obtaining the priority weights from consistent or inconsistent interval fuzzy preference relations. Lan et al. [21] proposed a method to derive interval weights by transforming a multiplicative consistent interval fuzzy preference relation into an additive consistent interval fuzzy preference relation. Xu and Chen [50] established some simple and practical linear programming models for deriving the priority weights from various interval fuzzy preference relations. Genc et al. [9] introduced the concept of interval multiplicative transitivity of an interval fuzzy

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preference relation and put forward a simple formula to derive the priority vector for consistent interval fuzzy preference relation based on the interval multiplicative transitivity.

In the preference relations based decision making problems, the measure of consistency level is of high importance. The lack of acceptable consistency can lead to a misleading solution. Most of the literature about IMPR focuses on how to obtain the weights or interval weights (priority vector) for ranking alternatives. Compared with other preference relations, the theories on how to measure and improve the consistency level of IMPR is relatively scarce. For example, Liu [23] defined the acceptably consistent interval reciprocal comparison matrix and proposed a convex combination method to obtain interval weights from an acceptably consistent interval reciprocal comparison matrix. Wang et al. [37] proposed a two-stage logarithmic goal programming method to generate weights from interval comparison matrices, which can be either consistent or inconsistent. By contrast, the consistency condition in Liu [23] is a little too stronger, where an IMPR is said to be of consistency or acceptable consistency only if both of its two corresponding multiplicative preference relations are of consistency or acceptable consistency and there is no complementary effect on consistency between these two corresponding multiplicative preference relations. On the contrary, the consistency condition in the model of Wang et al. [37] is a little too weaker. An IMPR with larger width elements is more inclined to consistent than an IMPR with smaller width elements and it is inconvenient to define a consistent level for an IMPR in the model of Wang et al. [37].

To facilitate experts the expression of consistent preferences in the decision processes, Herrera-Viedma et al. [12] presented a characterization of the consistency property defined by the additive or multiplicative transitivity property of the fuzzy preference relations, by which a method is designed to construct consistent multiplicative preference relations from a set of $n-1$ preference values. In this paper, applying this method, we define and derive a referenced consistent IMPR for a given IMPR, by which the consistency level of the IMPR can be measured and a straightforward approach to improve the consistency of the IMPR can also be easily derived. To do this, the rest of this paper is organized as follows. In Section 2, a brief introduction to the basic notions is provided. Section 3 proposed a new method to measure the consistency level of IMPR. In Section 4, a consistency model for GDM problems with IMPR is proposed. Section 5 makes a comparison between different consistency models of IMPR. Section 6 gives the conclusions.

2. Preliminaries

Let $X=(x_1, x_2, \ldots, x_n)$ be a finite set of alternatives and $I=\{1, 2, \ldots, n\}$ be the set of index. Saaty [28] proposed the multiplicative preference relation to describe an expert's preference, which was defined as following:

Definition 1. [28]. A multiplicative preference relation R on a set of alternatives X is represented by a reciprocal matrix $R=(r_{ij})_{n\times n}$ with

$$r_{ij} > 0, \quad r_{ij}r_{ji} = 1, \quad r_{ii} = 1, \quad \forall i, \quad j \in I \tag{1}$$

where r_{ij} expresses the ratio of the preference intensity of alternative x_i to that of x_j. Especially, $r_{ij}<1$ indicates that x_j is preferred to x_i; $r_{ij}=1$ indicates indifference between x_i and x_j; $r_{ij}>1$ indicates that x_i is preferred to x_j.

Since Saaty's 1–9 scale [28] is the most widely used tool to deal with such a situation, unless stated otherwise, in what follows, we apply the 1–9 ratio scale to express the preference values. That is, $r_{ij}=1/9$ indicates that x_j is absolutely preferred to x_i and $r_{ij}=9$ indicates that x_i is absolutely preferred to x_j.

Definition 2. [31]. A multiplicative preference relation $R=(r_{ij})_{n\times n}$ is consistent, if the following transitivity is satisfied

$$r_{ij} = r_{il}r_{lj}, \quad \forall i, \quad j, \quad l \in I. \tag{2}$$

Definition 3. [31]. An IMPR $\tilde{R}$ on a set of alternatives X is represented by a reciprocal matrix

$$\tilde{R} = (\tilde{r}_{ij})_{n\times n} = \begin{bmatrix} [1,1] & [r_{12}^-, r_{12}^+] & \cdots & [r_{1n}^-, r_{1n}^+] \\ [r_{21}^-, r_{21}^+] & [1,1] & \cdots & [r_{2n}^-, r_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [r_{n1}^-, r_{n1}^+] & [r_{n2}^-, r_{n2}^+] & \cdots & [1,1] \end{bmatrix}$$

where $0 < r_{ij}^- \le r_{ij}^+, r_{ij}^- r_{ji}^+ = 1, r_{ij}^+ r_{ji}^- = 1$, and $\tilde{r}_{ij}$ indicates the ratio of the preference intensity of alternative x_i to that of x_j is between r_{ij}^- and r_{ij}^+.

In addition, Liu [23] gave the following definition of consistency for an IMPR, which this paper applies to define a consistent IMPR.

Definition 4. [23]. Let $\tilde{R}=(\tilde{r}_{ij})_{n\times n}$ be an IMPR. $\tilde{R}$ is said to be consistent if the multiplicative preference relations $R^-=(R_{ij}^-)_{n\times n}$ and $R^+=(R_{ij}^+)_{n\times n}$ are both consistent, where

$$R_{ij}^- = \begin{cases} r_{ij}^+, i<j \\ 1, i=j \\ r_{ij}^-, i>j \end{cases} \quad \text{and} \quad R_{ij}^+ = \begin{cases} r_{ij}^-, i<j \\ 1, i=j \\ r_{ij}^+, i>j \end{cases}. \tag{3}$$

If the multiplicative preference relations $R^-=(R_{ij}^-)_{n\times n}$ and $R^+=(R_{ij}^+)_{n\times n}$ are all of acceptable consistency, then $\tilde{R}=(\tilde{r}_{ij})_{n\times n}$ is acceptably consistent. Otherwise, $\tilde{R}=(\tilde{r}_{ij})_{n\times n}$ is said to be unacceptably consistent [23].