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A quantitative comparison of stochastic mortality models on Italian population data

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ABSTRACT

Mortality models play a basic role in the evaluation of longevity risk by demographers and actuaries. Their performance strongly depends on the different patterns shown by mortality data in different countries. A comprehensive quantitative comparison of the most used methods for forecasting mortality is presented, aimed at evaluating both the goodness of fit and the forecasting performance of these mortality models on Italian demographic data. First, the classical Lee–Carter model is compared to some generalizations that change the order of Singular Value Decomposition approximation and include cohort effects. Then one-way and two-way functional data approaches are considered. Such an analysis extends the current literature on Italian mortality data, on both the number of considered models and their rigorous assessment. Results indicate that generally functional models outperform the classical ones; unfortunately, even if the cohort effect is quite substantial, a suitable procedure for its robust and efficient evaluation is yet to be proposed. To this end, a viable correction for cohort effects is suggested and its performance tested on some of the presented models.

1. Introduction

Population ageing is a global phenomenon that has profound implications for many facets of human life. Even if the pace of change differs greatly among different countries, there is a general trend: the older population is growing faster than the total population in practically all regions of the world, and the difference in growth rates is increasing. Thus, the need for improved information and analysis of demographic ageing also increases. Knowledge is essential to assist policy makers in defining, formulating and evaluating goals and programmes, and to raise public awareness and support for needed policy changes. Indeed, the United Nations released new population projections for all countries in July 2014. Gerland et al. (2014) analysed the data and described the probabilistic population projections for the entire world as well as individual regions and countries. According to their analysis, world population is likely to continue growing for the rest of the century; furthermore, the ratio of working-age people to older people is almost certain to decline substantially in all countries, not just currently developed ones.

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In such a context, the study of human mortality data, aimed at both identifying patterns in recorded data and predicting their future evolution, plays a central role; assessing trends in mortality is also a fundamental issue in actuarial modelling, because of the impact of extended longevity on pricing and reserve allocations as well as solvency assessment in life office and pension plans. Such a great focus on longevity risk has led to a large number of new mortality models in the last twenty years. Many authors thoroughly investigated and extended the model proposed by Lee and Carter in their seminal work (Lee and Carter, 1992), now considered a milestone in the stochastic modelling and forecasting of trends in mortality data. Among them, Renshaw and Haberman (2003) proposed assuming a Poisson distribution of deaths and the incorporation of a cohort effect (Renshaw and Haberman, 2006); Currie et al. (2004) suggested the use of B-splines and P-splines to fit the mortality surface; Hyndman and Ullah (2007) smoothed data and used functional principal components; more recently, Dokumentov and Hyndman (2014) and Huang et al. (2009) proposed regularization approaches for bivariate functional data. These models differ in a number of key elements, such as sources of randomness, assumptions on smoothness, inclusion of cohort effects and estimation method: however, as many comparison studies confirm (Cairns et al., 2009; Dowd et al., 2010), none of them clearly dominate the others under all the considered comparison criteria. Moreover, models that fit historical data well can still give inadequate forecasts, so the choice of a model cannot be based only on its ability to explain historical patterns of mortality. Finally, models that are the best for one country might not be as suitable for another, due to differences in the evolution of each country's mortality rates.

In particular, for the Italian population demographic data, D'Amato et al. (2011) performed a comparative assessment between the basic Lee–Carter Model (Lee and Carter, 1992) and the Functional Demographic Model by Hyndman and Ullah (2007), presenting empirical results based on graphical analyses. The present study builds on this work in proposing a broader and quantitative comparison of the main mortality models (including the ones considered therein), for evaluating objectively both their goodness of fit and their forecasting performance on Italian population data.

Assuming as a basic reference the classical Lee–Carter model (LC), we first test the results obtained by introducing changes in noise modelling, next we test the effect of changing the order of SVD approximations and of including cohort effects. Finally we shift to one-way and two-way functional data approaches. In Section 2 we describe the models considered in the comparison and suggest a strategy to apply a specific correction for the cohort effect; in Section 3 we evaluate qualitatively and quantitatively the complexity and goodness of fit of all these models and in Section 4 we discuss their forecasting performance by means of a backtesting procedure. Some concluding remarks are reported in Section 5.

2. Mortality models

In the 20th century, human mortality globally declined over time in developed countries. The resulting life expectancy increase is explained by two events: an increasing concentration of deaths around the mode (at old ages) of the curve of deaths and the shift of the maximal age at death to the right, technically referred to as *rectangularization* and *expansion* of the survival curve, respectively. Moreover, higher levels of accidental deaths at young ages (the so called "young mortality hump"), with corresponding larger dispersion, have been observed recently.

Accounting for the evolution in time of mortality implies the use of "projected" survival models aiming at describing future age patterns of mortality, on the basis of the experienced mortality trend. When working with projected survival models a dynamic approach to mortality is required, expressing mortality as a function of both the age x and the calendar year t. The force of mortality (or mortality intensity) represents the instantaneous rate of mortality at any given integer age x during the calendar year t. In reliability theory, this concept is usually referred to as the failure rate or the hazard function. It is formally defined as follows:

$$\mu_x(t) = \lim_{\Delta \to 0} \frac{P[x < T_0(t-x) \le x + \Delta | T_0(t-x) > x]}{\Delta}, \tag{1}$$

where $T_0(t-x)$ is the time to death of an individual born in year $t-x$. Under the assumption

$$\mu_{x+\xi_1}(t+\xi_2) = \mu_x(t), \quad \text{for } 0 \le \xi_1, \xi_2 < 1,$$

the maximum likelihood estimate $\tilde{\mu}_x(t)$ of the force of mortality is given by:

$$\tilde{\mu}_x(t) = \frac{D_{xt}}{E_{xt}} = m_x(t), \tag{2}$$

where D_{xt} is the number of deaths recorded at age x last birthday during the calendar year t. E_{xt} is the exposure-to-risk in the age interval $[x, x+1)$ during calendar year t. This is an estimate of the population exposed to the risk of death during this age–time interval. The central death rate (or mortality rate) $m_x(t) \equiv m_{x,t}$ is referred to as crude (i.e. unsmoothed) death rate, being the proportion of people of a given age expected to die within the year, expressed in terms of the expected number of life-years rather than in terms of the number of individuals initially present in a group.

As clearly stated by several authors (Currie, 2016; Hunt and Blake, 2014), in an effort to produce a unified description of the plethora of existing mortality models, many of them can be expressed in the language of generalized (non)linear models to account for the interaction between the age x, the calendar year (or period) t and the year of birth (or cohort) $y = t - x$.