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Good characterizations and linear time recognition for 2-probe block graphs[☆]

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ABSTRACT

Block graphs are graphs in which every block (biconnected component) is a clique. A graph $G = (V, E)$ is said to be an (unpartitioned) k-probe block graph if there exist k independent sets $\mathbb{N}_i \subseteq V$, $1 \le i \le k$, such that the graph G' obtained from G by adding certain edges between vertices inside the sets $\mathbb{N}_i$, $1 \le i \le k$, is a block graph; if the independent sets $\mathbb{N}_i$ are given, G is called a partitioned k-probe block graph. In this paper we give good characterizations for 2-probe block graphs, in both unpartitioned and partitioned cases. As an algorithmic implication, partitioned and unpartitioned probe block graphs can be recognized in linear time, improving a recognition algorithm of cubic time complexity previously obtained by Chang et al. (2011).

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1. Introduction

Given a graph class $\mathcal{C}$, a graph $G = (V, E)$ is called a *probe $\mathcal{C}$ graph* if there exists an independent set $\mathbb{N} \subseteq V$ (of *non-probes*) and a set $E' \subseteq \binom{\mathbb{N}}{2}$ such that the graph $G' = (V, E \cup E')$ is in the class $\mathcal{C}$, where $\binom{\mathbb{N}}{2}$ stands for the set of all 2-element subsets of $\mathbb{N}$. A graph $G = (V, E)$ with a *given* independent set $\mathbb{N} \subseteq V$ is said to be a *partitioned probe $\mathcal{C}$ graph* if there exists a set $E' \subseteq \binom{\mathbb{N}}{2}$ such that the graph $G' = (V, E \cup E')$ is in the class $\mathcal{C}$. In both cases, G' is called a *$\mathcal{C}$ embedding* of G. Thus, a graph is a (partitioned) probe $\mathcal{C}$ graph if and only if it admits a $\mathcal{C}$ embedding.

Recognizing partitioned probe $\mathcal{C}$ graphs is a special case of the $\mathcal{C}$-GRAPH SANDWICH problem (cf. [9]). More precisely, given two graphs $G_i = (V, E_i)$, $i = 1, 2$, on the same vertex V such that $E_1 \subseteq E_2$, the $\mathcal{C}$-GRAPH SANDWICH problem asks for the existence of a graph $G = (V, E)$ such that $E_1 \subseteq E \subseteq E_2$ and G is in $\mathcal{C}$. Recognizing partitioned probe $\mathcal{C}$ graphs with a given independent set $\mathbb{N}$ is a special case of the $\mathcal{C}$-GRAPH SANDWICH problem, where $E_2 \setminus E_1 = \binom{\mathbb{N}}{2}$. Both concepts stem from computational biology; see, *e.g.*, [8,9,18,19].

Probe graphs have been investigated for various graph classes; see [3] for more information.

Recently, the concept of probe graphs has been generalized in [4]. A graph G is said to be a *k-probe $\mathcal{C}$ graph* if there exist independent sets $\mathbb{N}_1, \dots, \mathbb{N}_k$ in G such that there exists a graph $G' \in \mathcal{C}$ (an *embedding* of G) such that for every edge xy in G' which is not an edge of G there exists an i with $x, y \in \mathbb{N}_i$. In the case $k = 1$, G is a *probe $\mathcal{C}$ graph.*

We refer to the partitioned case of the problem when a collection of independent sets $\mathbb{N}_i$, $i = 1, \dots, k$, is a part of the input; otherwise, it is an unpartitioned case. For historical reasons we call the set of vertices $\mathbb{P} = V \setminus \bigcup_{i=1}^{k} \mathbb{N}_i$ the set of *probes* and the vertices of $\bigcup_{i=1}^{k} \mathbb{N}_i$ the set of *non-probes*.

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In [4], k-probe complete graphs and k-probe block graphs have been investigated. The authors proved that, for fixed k, k-probe complete graphs can be characterized by finitely many forbidden induced subgraphs, their proof is however not constructive. They also showed, implicitly, that k-probe complete graphs and k-probe block graphs can be recognized in cubic time. The case $k = 1$, *e.g.*, probe complete graphs and probe block graphs, has been discussed in depth in [15].

In this paper, we study 2-probe complete graphs and 2-probe block graphs in more detail. Our main results are:

- A characterization of partitioned 2-probe block graphs in terms of certain "enhanced graph" (Theorem 5), stating that G is a partitioned 2-probe block graph if and only if the enhanced graph G^* is a block graph.
- Forbidden induced subgraph characterizations of unpartitioned 2-probe block graphs (Theorem 6).
- Linear time recognition for 2-probe block graphs, in both partitioned and unpartitioned cases.

The first result is of great interest because the enhanced graph contains only necessary edges, *i.e.*, new edges that must be added. In this sense, the enhanced graph is an optimal embedding of the probe graph. This type of characterization is rarely possible, and our result is the first one in case $k = 2$. In case of probe graphs, *i.e.*, $k = 1$ only few are known: In [1] it is shown that a graph is a partitioned probe threshold graph, respectively, a partitioned probe trivially perfect graph if and only if a certain enhanced graph is a threshold graph, respectively, a trivially perfect graph. In [14] it is shown that a graph is a partitioned chain graph if and only if a certain enhanced graph is a chain graph, and recently, [15] (cf. Theorem 1) proved that a graph is a partitioned block graph if and only if a certain enhanced graph is a block graph. For some other cases, a certain enhanced graph can be defined that admits some nice properties; see [5,10,18].

Forbidden induced subgraph characterizations are very desirable as they (or their proofs) often imply polynomial time for recognition, and give a lot of structural information of the graphs.[1] This is the case with the second result. Based on our forbidden induced subgraph characterization, we will obtain a linear time algorithm for recognizing if a given graph is a 2-probe block graph, improving the cubic time complexity provided previously in [4].

The paper is structured as follows. In Section 2, we collect all the necessary definitions, and review results about probe complete graphs and probe block graphs. In Section 3, we discuss 2-probe complete graphs. Partitioned and unpartitioned 2-probe block graphs will be considered in Section 4 and in Section 5, respectively. A linear time recognition algorithm of unpartitioned 2-probe block graphs is proposed in Section 6. We conclude the paper with some open problems in Section 7.

2. Definitions and notion

In a graph, a set of vertices is an *independent set*, respectively, a *clique* if no two, respectively, every two vertices in this set are adjacent. For two graphs G and H, we write $G + H$ for the disjoint union of G and H, and $2G$ for $G + G$. The *join* $G \star H$ is obtained from $G + H$ by adding all possible edges xy between any vertex x in G and any vertex y in H. The complete graph with n vertices is denoted by K_n. The path and cycle with n vertices of length $n - 1$, respectively, of length n, is denoted by P_n, respectively, C_n. Let $G = (V, E)$ be a graph. For a vertex $v \in V$ we write $N(v)$ for the set of its neighbors in G. A *universal* vertex v is one such that $N(v) \cup \{v\} = V$. For a subset $U \subseteq V$ we write $G[U]$ for the subgraph of G induced by U and $G - U$ for the graph $G[V \setminus U]$; for a vertex v we write $G - v$ rather than $G[V \setminus \{v\}]$.

A (connected or not) graph is a *block graph* if each of its maximal 2-connected components, *i.e.*, its blocks, is a clique. A *chordal graph* is one in which every cycle C_ℓ of length $\ell \geq 4$ has a chord. (A chord of a cycle is an edge not belonging to the cycle but joining to vertices of the cycle.) A *diamond* is the complete graph on four vertices minus an edge. It is well-known (and easy to see) that block graphs are exactly the chordal graphs without induced diamond.

Proposition 1 (*Folklore*). *A graph is a block graph if and only if it is a diamond-free chordal graph.*

Here, given a graph F, a graph is said to be F*-free* if it has no induced subgraph isomorphic to F. For a set of graphs $\mathcal{F}$, a graph is said to be $\mathcal{F}$*-free* if it is F-free for each $F \in \mathcal{F}$.

A graph G is called *distance-hereditary* if for all vertices $u, v \in V(G)$ any induced path between u and v is a shortest path. A graph G is called *ptolemaic* if, in any connected component of G, every four vertices satisfy the so-called *ptolemaic inequality* (cf. [11]).

Proposition 2 (*Folklore*).

(i) *Ptolemaic graphs, gem-free chordal graphs, and C_4-free distance-hereditary graphs coincide.*
(ii) *Distance-hereditary graphs are exactly the graphs without induced house, hole, domino, gem.*

Here, a *house* is a 5-cycle with exactly one chord, a *hole* is a C_ℓ, $\ell \geq 5$, a *domino* is a 6-cycle with exactly one long chord, and a *gem* is the join $P_4 \star K_1$.

Another graph class that will be important in our discussion is the class of P_4-free graphs, or *cographs*. Clearly, by Proposition 2, cographs are distance-hereditary, and we will often use the following well known fact.

[1] That is why characterizing probe interval graphs by forbidden induced subgraphs is a long-standing interesting open problem; see [16]

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