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Open shop scheduling problems with conflict graphs

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ABSTRACT

This paper deals with open shops subject to the constraint that some conflicting jobs cannot be processed simultaneously on different machines. In the context of our study, these conflicts are given by an undirected graph, called the conflict graph. We seek a schedule that minimizes the maximum completion time (makespan). We first prove the NP-hardness of different versions of this problem. Then, we present polynomial-time solvable cases, for which we devise simple and efficient algorithms. On the other hand, we present heuristics and lower bounds for the general case. The heuristics are based on the construction of matchings in bipartite graphs as well as on a specific insertion technique combined with beam search. Finally, we test the efficiency of the heuristics and report our computational experiments that display satisfying results.

1. Introduction

We consider the following scheduling problem: given are a set $\{J_j, j = 1, \dots, n\}$ of n jobs and a set $\{M_i, i = 1, \dots, m\}$ of m machines. Each job J_j consists of m operations J_{ij} $(i = 1, \dots, m)$, where J_{ij} has to be processed on machine M_i for p_{ij} time units. The processing times p_{ij} are nonnegative numbers and some of them may be null. The operations of each job can be processed in any order, but only one at a time. Each machine can process at most one operation at a time. We assume that there is a simple and undirected graph $G = (V, E)$ over the jobs, called the conflict graph. Each vertex in G represents a job and jobs that are represented by adjacent vertices in G are conflicting and cannot be processed simultaneously on different machines. Therefore, if J_l and J_k are in conflict, then the time intervals assigned to the operations J_{ul} and J_{vk}, with $u \neq v$, do not overlap. We seek a schedule that minimizes the maximum completion time (C_{max}), also called the makespan. This problem is referred to as Open Shop scheduling with Conflict graph (OSC) and can be solved in polynomial time if G is a clique (all the operations are processed in disjoint time intervals and $C_{max} = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij}$), or if G is an independent set and the jobs are processed on two machines (the problem is reduced to the basic two-machine open shop problem and can be solved using the Longest Alternate Processing Time first (LAPT) rule proposed in [23] or an algorithm proposed earlier in [18]). The graph $\overline{G} = (V, \overline{E})$ (called the agreement graph) denotes the complement of the conflict graph $G = (V, E)$. It is clear that the open shop scheduling with conflict graph and the open shop scheduling with agreement graph are polynomially equivalent.

The OSC problem can be found in the open shop scheduling with renewable resources. In this latter problem, we have a set $\{R_s, s = 1, \dots, r\}$ of r additional renewable resources, each resource R_s is discretely divisible and its usage at any time is constrained by an available amount $|R_s|$. Each operation J_{ij} requires $R_s(J_{ij})$ units of resource R_s. In this situation, the conflicts arise between operations *rather* than jobs, in general, because the operations of a job may have different requirements of a given resource. Let us consider the following example of two jobs J_l and J_k to be processed on two machines. Only one resource R_s is required for the execution of the jobs such that: $|R_s| = 1$, $R_s(J_{1l}) = R_s(J_{1k}) = R_s(J_{2k}) = 1$ and $R_s(J_{2k}) = 0$. It

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follows that J_{1l} and J_{2k} are in conflict but J_{2l} and J_{1k} are not. Thus, this problem cannot be modeled by a conflict graph over the jobs.

If we assume that the operations of each job have the same requirement of each resource, then a conflict arises between a subset of jobs if the total requirement of at least one resource exceeds its capacity. In general, this setting can be modeled by a conflict hypergraph. Indeed, consider the following example of 3 jobs $\{J_1, J_2, J_3\}$ to be processed on three machines. Suppose that we have one renewable resource R_s such that $|R_s| = 2$ and $R_s(J_{ij}) = 1; i = \overline{1, 3}, j = \overline{1, 3}$. Hence, J_1 and J_2 are not in conflict. The same for J_1 and J_3 and also for J_2 and J_3. The conflict graph is then an independent set, which means that all the jobs can be processed simultaneously. This is not possible since the total requirement of all the jobs exceeds the resource capacity. Therefore, this case requires a conflict hypergraph. However, in special cases a conflict graph suffices, e.g. if the number of machines equals two (an example will be given in the first paragraph of Section 4) and a more general case will be discussed in Section 3. Now, if $m = 2$ and $\max_{k,l;k\neq l}\{R_s(J_{1k}) + R_s(J_{2l})\} \leq |R_s|, \forall R_s$, i.e. the available amount of each resource is sufficient to process the operations on the two machines, then there are no conflicts between the jobs and the conflict graph associated is an independent set (that is $O2 \parallel C_{max}$).

Blazewicz et al. [6] expanded the three field classification, $\alpha/\beta/\gamma$, to cover the resource constrained scheduling problems. Parameter β denotes the job and resource characteristics. The presence of additional resources is specified by $res\lambda\sigma\delta$, where $\lambda, \sigma, \delta \in \{., k\}$ represent respectively the number of resource types, resource availabilities and resource requirements. A dot "." indicates that the corresponding parameter can take any integer value, whereas a positive integer k indicates that the number of resource types is equal to k, each resource is available in the amount of k units and the resource requirements of each operation are equal to at most k units. We denote by $res^t\lambda\sigma\delta$ the case in which the operations of each job J_j require the same amount of each resource R_s and we refer to this requirement by $R_s(J_j)$, i.e. $R_s(J_{1j}) = \cdots = R_s(J_{mj}) = R_s(J_j), s = 1, \ldots, r, j = 1, \ldots, n$. Observe that the scheduling problem $\alpha/res^t\lambda\sigma\delta/\gamma$ is a subproblem of $\alpha/res\lambda\sigma\delta/\gamma$.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents NP-hardness results while Section 4 discusses well solvable cases. Section 5 presents three lower bounds. Section 6 presents the heuristics. Section 7 is devoted to the experimental study. Section 8 draws the main conclusions and perspectives.

2. Related literature

Scheduling with Conflict graphs has been studied on Identical machines, the inputs consist of n jobs that have to be processed on m identical machines and a conflict graph G over the jobs, such that adjacent jobs in G are conflicting and cannot be processed simultaneously on different machines, this problem is denoted by SCI. For two machines and arbitrary processing times, the SCI problem is NP-hard even if the conflict graph is an independent set [17]. Baker and Coffman [1] have called Mutual Exclusion Scheduling (MES) the SCI problem in which each job requires one unit of processing time. This problem corresponds exactly to a minimum coloring of the conflict graph G such that each color appears at most m times. The MES problem is NP-hard since the problem of finding a minimum coloring of a graph is NP-hard [21]. However, there are many special graph classes on which the MES problem is solvable in polynomial time. For more results about the MES problem, the interested reader is referred to the papers [2,16]. The authors of [15] provided an exact algorithm for the SCI problem on two identical machines and jobs sizes in $\{1, 2\}$. They also showed that for jobs sizes in $\{1, 2, 3, 4\}$ the problem is APX-hard and they left the question of the exact status of the case of jobs sizes in $\{1, 2, 3\}$ as an open problem. However, in [2] the authors established that this open problem is NP-hard in the strong sense for complements of bipartite graphs. Lately, the same authors closed definitely in [4] the complexity status of the SCI problem on two machines with two fixed processing times. Indeed, they proved that for two machines and jobs sizes in $\{a, 2a + b\}$ ($a \geq 1$ and ($b \geq 1$ or $-a < b < 0$)), the SCI problem is strongly NP-hard when restricted to complements of bipartite graphs. Furthermore, they reported that the SCI problem is polynomially equivalent to a special case of the resource-constrained scheduling problem, from which new complexity results of this latter have been derived. Then, they extended in [3] the NP-hardness results of the SCI problem presented in [4] to more general agreement graphs and they established new complexity results for the case of split and complement of bipartite agreement graphs. In [7], the authors considered a complementary problem called scheduling incompatible jobs (jobs that cannot be processed by the same machine). They modeled these constraints using an undirected graph, in which each edge joins a pair of incompatible jobs. They gave polynomial time approximation algorithms, with constant worst-case guarantee, when restricted to particular graph classes (trees, bipartite, bounded-treewidth and complete graphs). In [27], the authors studied the problem of scheduling with conflict graph in a flow shop system with unit-time operations. They developed mathematical models and a branch and bound algorithm alongside with an experimental study.

Since our problem is related to the open shop scheduling under resource constraints, we focus in the following review of the literature on this problem. Blazewicz et al. [6,5] have presented an overview and complexity classification for the case of renewable resources (Table 1). In [13,11,12], the authors have studied the preemptive version of the m-machine open shop problem with renewable and nonrenewable resources, they have formulated the problem in terms of edge coloring in a bipartite multigraph and they have exhibited a polynomial algorithm for the case where the curve of the resource availability is of staircase type. Jurisch and Kubiak [20] have considered the two-machine open shop with renewable resources. They first showed that the preemptive case can be solved in $O(n^3)$ through a reduction to the max-flow problem, then they converted the preemptive solution into a non-preemptive schedule when the number of resources is limited to one. They also proved the NP-hardness of two particular cases (Table 1). Tautenhahn and Woeginger [26] have studied the open shop