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Cutting out polygon collections with a saw[☆]

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A B S T R A C T

(I) Given a segment-cuttable polygon P drawn on a planar piece of material Q, we show how to cut P out of Q by a (short) segment saw with a total length of the cuts no more than 2.5 times the optimal. We revise the algorithm of Demaine et al. (2001) so as to achieve this ratio.

(II) We prove that any collection $\mathcal{R}$ of n disjoint[1] axis-parallel rectangles drawn on a planar piece of material Q is cuttable by at most $4n$ rays and present an algorithm that runs in $O(n \log n)$ time for computing a suitable cutting sequence. In particular, the same result holds for cutting with an arbitrary segment saw (of any length).

(III) Given a collection $\mathcal{P}$ of segment-cuttable polygons drawn on a planar piece of material such that no two polygons in $\mathcal{P}$ touch each other, $\mathcal{P}$ is always cuttable by a sufficiently short segment saw. We also show that there exist collections of disjoint polygons that are uncuttable by a segment saw.

(IV) Given a collection $\mathcal{P}$ of disjoint polygons drawn on a planar piece of material Q, we present a polynomial-time algorithm that computes a suitable cutting sequence to cut the polygons in $\mathcal{P}$ out of Q using ray cuts when $\mathcal{P}$ is ray-cuttable and otherwise reports $\mathcal{P}$ as uncuttable.

1. Introduction

The problem of efficiently cutting out a simple polygon P drawn on a planar piece of material (such as wood, paper, glass) Q, was introduced by Overmars and Welzl in their seminal paper [20] from 1985. Since then, the problem has attracted the interest of many computational geometers.

A saw cut may split (divide) Q into a number of pieces—those that lie left of the cut and those that lie right of the cut. In some situations, the saw may stop short of splitting Q, in which case, the material remains as one solid piece. In any case we do not allow a cut to run through the interior of P. Several variants have been studied, primarily depending on the cutting tools used [1,3,5,6,9,11–14,20,23]: line cuts, ray cuts and segment cuts; they are described subsequently. In saw cutting, i.e., in all three models above, turns are impossible. The type of tool used in cutting determines the class of polygons that can be cut within that model.

The measures of efficiency commonly considered in polygon cutting are the total length of the cuts and the total number of cuts. Polygon cutting problems are useful in industry applications such as metal sheet cutting, paper cutting, furniture

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1 For brevity, a *collection of disjoint polygons* refers to a collection of polygons with pairwise disjoint interiors.

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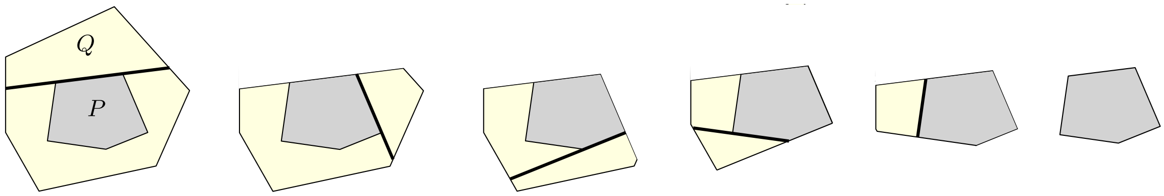

Fig. 1. Cutting a convex polygon P out of Q using 5 line cuts.

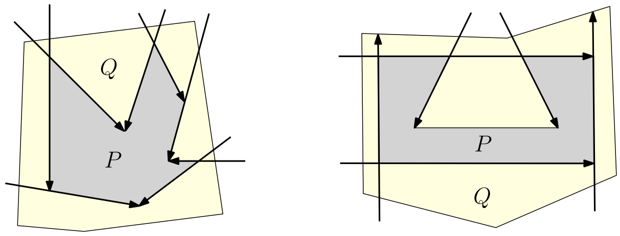

Fig. 2. Left: a ray-cuttable polygon. Right: a polygon which is not ray-cuttable or segment-cuttable.

manufacturing and numerous other areas of engineering, where smart cutting techniques with high efficiency may result in the reduction of production costs. For instance, reducing the total length of the cuts may result in lesser power requirement and extend the life of the cutting tool. Similarly, reducing the total number of cuts may save cutting time and extend the life of the cutting tool.

1.1. *Line cuts, ray cuts, and segment cuts*

A *line cut* (also called a *guillotine cut*) is a line that does not go through P and divides the current piece of material containing P (initially Q) into two pieces. For cutting P out of Q by line cuts, P must be convex. An instance of a cutting sequence using line cuts appears in Fig. 1. The most studied efficiency measure for line cutting is the total length of the cuts and several approximation algorithms have been obtained [1,3,5,6,9,11–13,20,23], including a PTAS proposed by Bereg et al. [3].

A *ray cut* comes from infinity and can stop at any point outside P, again, not necessarily splitting the piece of material into pieces. Ray cuts are usually used to cut out non-convex polygons; however, not all non-convex polygons can be cut by ray cuts. The following observation gives a necessary and sufficient condition for ray-cuttability, in the case of a single polygon; see Fig. 2. Several approximation algorithms can be found in [6,9,11,23].

Observation 1. *A polygon P drawn on a planar material is ray-cuttable if and only if every edge of P that has some material adjacent to it, can be extended to infinity from one of its endpoints without passing through the interior of P.*

A *segment cut* is similar to a ray cut, but is not required to start at infinity, it may start at some finite point. The *segment saw* (also referred to as *circular saw* in [11]) is abstracted as a line segment, which cuts through material when moved along its supporting line. Before executing a segment cut, the saw needs to be placed. A small example of a cutting sequence appears in Fig. 3. Recall that saw turns are impossible during a cut; however, if a small free space within Q is available, a segment cut can be initiated there by maneuvering (i.e., rotating) the saw. The space required for maneuvering the saw is proportional to the length of the saw. The problem of cutting a polygon by a segment saw was introduced by Demaine et al. [11] in 2001. The authors gave a characterization of the class of polygons cuttable by a (possibly short) segment saw: a polygon is cuttable in this model, i.e., by a sufficiently short segment saw, if and only if it does not have two adjacent reflex vertices (with interior angle $> \pi$).

Note that ray-cuttability is not equivalent with segment-cuttability. For instance the polygon P in Fig. 3 is segment-cuttable but not ray-cuttable; indeed, the condition specified by Observation 1 is not fulfilled by two edges of P at the bottom of the pocket.

For ease of analysis the length of the saw is assumed to be arbitrarily small, i.e., the segment abstracting the saw is as short as needed. Consequently, a segment cut can be initiated from an arbitrarily small available free space. In this model, several parts may result after a cut is made, and any of them can be removed (lifted) from the original plane. Moreover, free space may appear within the pieces of material from where future segment cuts can be initiated. The cutting process may continue independently on any of the separated pieces of material, if resulting parts contain subcollections of a larger collection to be cut out.

1.2. *Our results and related work*

Demaine et al. [11] presented an algorithm for cutting P out of its convex hull using segment cuts with a total number of cuts and total length of the cuts within constant factors of the respective optima. With regard to the total number of cuts,

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