ARTICLE IN PRESS

Discrete Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Approximability of guarding weak visibility polygons

Pritam Bhattacharya [a,*], Subir Kumar Ghosh [b], Bodhayan Roy [c]

[a] *Department of Computer Science and Engineering, Indian Institute of Technology, Kharagpur - 721302, India*

[b] *Department of Computer Science, Ramakrishna Mission Vivekananda University, Belur Math, Howrah - 711202, India*

[c] *Department of Computer Science and Engineering, Indian Institute of Technology, Powai, Mumbai - 400076, India*

ARTICLE INFO

Article history:
Received 29 June 2015
Received in revised form 29 September 2016
Accepted 19 December 2016
Available online xxxx

Keywords:
Art gallery problem
Visibility
Vertex guarding
Constant factor approximation algorithm
Inapproximability
Euclidean shortest path tree

ABSTRACT

The art gallery problem enquires about the least number of guards that are sufficient to ensure that an art gallery, represented by a polygon P, is fully guarded. In 1998, the problems of finding the minimum number of point guards, vertex guards, and edge guards required to guard P were shown to be APX-hard by Eidenbenz, Widmayer and Stamm. In 1987, Ghosh presented approximation algorithms for vertex guards and edge guards that achieved a ratio of $\mathcal{O}(\log n)$, which was improved up to $\mathcal{O}(\log \log OPT)$ by King and Kirkpatrick (2011). It has been conjectured that constant-factor approximation algorithms exist for these problems. We settle the conjecture for the special class of polygons that are weakly visible from an edge and contain no holes by presenting a 6-approximation algorithm for finding the minimum number of vertex guards that runs in $\mathcal{O}(n^2)$ time. On the other hand, for weak visibility polygons with holes, we present a reduction from the Set Cover problem to show that there cannot exist a polynomial time algorithm for the vertex guard problem with an approximation ratio better than $((1-\epsilon)/12)\ln n$ for any $\epsilon > 0$, unless $NP = P$. We also show that, for the special class of polygons without holes that are orthogonal as well as weakly visible from an edge, the approximation ratio can be improved to 3. Finally, we consider the point guard problem and show that it is NP-hard in the case of polygons weakly visible from an edge.

1. Introduction

1.1. The art gallery problem and its variants

The art gallery problem enquires about the least number of guards that are sufficient to ensure that an art gallery (represented by a polygon P) is fully guarded, assuming that a guard's field of view covers 360° as well as an unbounded distance. This problem was first posed by Victor Klee in a conference in 1973, and in the course of time, it has turned into one of the most investigated problems in computational geometry.

A *polygon* P is defined to be a closed region in the plane bounded by a finite set of line segments, called edges of P, such that, between any two points of P, there exists a path which does not intersect any edge of P. If the boundary of a polygon P consists of two or more cycles, then P is called a *polygon with holes* (see Fig. 1). Otherwise, P is called a *simple polygon* or a *polygon without holes* (see Fig. 2).

An art gallery can be viewed as an n-sided polygon P (with or without holes) and guards as points inside P. Any point $z \in P$ is said to be *visible* from a guard g if the line segment zg does not intersect the exterior of P (see Figs. 1 and 2). In

* Corresponding author.
E-mail addresses: pritam.bhattacharya@cse.iitkgp.ernet.in (P. Bhattacharya), ghosh@tifr.res.in (S.K. Ghosh), bodhayan.roy@gmail.com (B. Roy).

http://dx.doi.org/10.1016/j.dam.2016.12.015

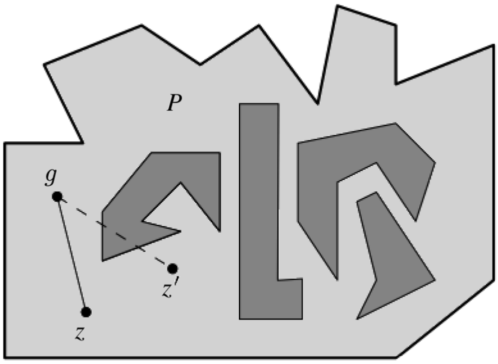

Fig. 1. Polygon with holes.

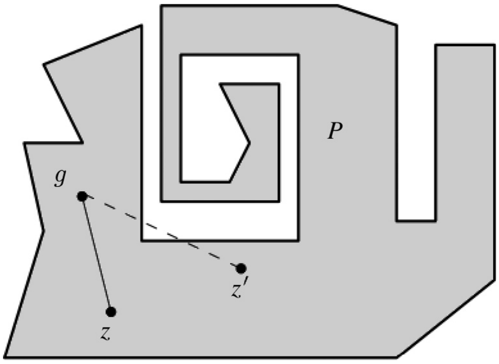

Fig. 2. Polygon without holes.

general, guards may be placed anywhere inside P. If the guards are allowed to be placed only on vertices of P, they are called *vertex guards*. If there is no such restriction, guards are called *point guards*. Point and vertex guards together are also referred to as *stationary guards*. If guards are allowed to patrol along a line segment inside P, they are called *mobile guards*. If they are allowed to patrol only along the edges of P, they are called *edge guards* [17,28].

In 1975, Chvátal [8] showed that $\lfloor \frac{n}{3} \rfloor$ stationary guards are sufficient and sometimes necessary (see Fig. 3) for guarding a simple polygon. In 1978, Fisk [15] presented a simpler and more elegant proof of this result. For a simple orthogonal polygon, whose edges are either horizontal or vertical, Kahn et al. [22] and also O'Rourke [27] showed that $\lfloor \frac{n}{4} \rfloor$ stationary guards are sufficient and sometimes necessary (see Fig. 4).

1.2. Related hardness and approximation results

The decision version of the art gallery problem is to determine, given a polygon P and a number k as input, whether the polygon P can be guarded with k or fewer guards. The problem was first proved to be NP-complete for polygons with holes by O'Rourke and Supowit [29]. For guarding simple polygons, it was proved to be NP-complete for vertex guards by Lee and Lin [25], and their proof was generalized to work for point guards by Aggarwal [1]. The problem is NP-hard even for simple orthogonal polygons as shown by Katz and Roisman [23] and Schuchardt and Hecker [30]. Each one of these hardness results hold irrespective of whether we are dealing with vertex guards, edge guards, or point guards.

In 1987, Ghosh [16,18] provided an $\mathcal{O}(\log n)$-approximation algorithm for the case of vertex and edge guards by discretizing the input polygon and treating it as an instance of the Set Cover problem. In fact, applying methods for the Set Cover problem developed after Ghosh's algorithm, it is easy to obtain an approximation factor of $\mathcal{O}(\log OPT)$ for vertex guarding simple polygons or $\mathcal{O}(\log h \log OPT)$ for vertex guarding a polygon with h holes. Deshpande et al. [9] obtained an approximation factor of $\mathcal{O}(\log OPT)$ for point guards or perimeter guards by developing a sophisticated discretization method that runs in pseudopolynomial time. Efrat and Har-Peled [11] provided a randomized algorithm with the same approximation ratio that runs in fully polynomial expected time. For guarding simple polygons using vertex guards and perimeter guards, King and Kirkpatrick [24] obtained an approximation ratio of $\mathcal{O}(\log \log OPT)$ in 2011.

In 1998, Eidenbenz, Stamm and Widmayer [12,13] proved that the problem is APX-complete, implying that an approximation ratio better than a fixed constant cannot be achieved unless $P = NP$. They also proved that if the input polygon is allowed to contain holes, then there cannot exist a polynomial time algorithm for the problem with an approximation ratio better than $((1-\epsilon)/12) \ln n$ for any $\epsilon > 0$, unless NP $\subseteq$ TIME$(n^{\mathcal{O}(\log \log n)})$. Contrastingly, in the case