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A heuristic for cumulative vehicle routing using column generation

Daya Ram Gaur [a], Rishi Ranjan Singh [b,*,1]

[a] Department of Mathematics and Computer Science, University of Lethbridge, 4401 University Drive, Lethbridge, Alberta, Canada

[b] Department of Information Technology, Indian Institute of Information Technology Allahabad, Jhalwa, Allahabad, Uttar Pradesh - 211012, India

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ABSTRACT

Cumulative vehicle routing problems are a simplified model of fuel consumption in vehicle routing problems. Here we computationally study, an inexact approach for constructing solutions to cumulative vehicle routing problems based on rounding solutions to a linear program. The linear program is based on the set cover formulation and is solved using column generation. The pricing subproblem is solved heuristically using dynamic programming. Simulation results show that a simple scalable strategy gives solutions with cost close to the lower bound given by the linear programming relaxation. We also give theoretical bounds on the integrality gap of the set cover formulation.

1. Introduction

Given a fleet of delivery vehicles at the depot, and customers with demands, the objective in the vehicle routing problem (VRP) [14,36] is to find a schedule for the vehicles, in order to meet the demands of the customers, that minimizes the total distance traveled by the vehicles. By considering additional constraints and requirements on the schedule, different variants of the problem are obtained. Typical constraints are capacity constraints on the vehicle, time windows constraints on the visits, and such. The variant of VRP that we study in this paper is known as the *cumulative vehicle routing problem* (Cu-VRP), as defined by Kara et al. [35]. In cumulative VRPs, the objective is to minimize the "cumulative" cost. The cumulative cost per unit distance is proportional to the weight of the empty vehicle plus the weight of the load.

Fuel cost is an important fraction of the transportation cost and depending on the medium, it can be as high as 60% (32% for cargo in the seas, 46% for the railroads and 60% for road transportation) [45]. Therefore, by minimizing the fuel consumption, we can reduce the entire transportation cost. The cumulative VRP is a simplified model for fuel consumption [47,35,49] in vehicle routing problems.

1.1. Cumulative VRPs

Next, we define the problem. The definitions and the notation are from Kara et al. [35], and Gaur et al. [26]. We are given a complete graph with non-negative weights on the edges. The edge weights satisfy the triangle inequality. A special node

* Corresponding author.
E-mail addresses: gaur@cs.uleth.ca (D.R. Gaur), rishi@iiita.ac.in (R.R. Singh).

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numbered 0 is the depot and all the client nodes are numbered from $\{1, 2, \ldots, n\}$. Stationed at the depot is a vehicle with capacity Q. There is a positive and integral demand d_i at each customer node i. We can think of d_i as the weight of the goods to be transported from the client node to the depot. The objective is to schedule the vehicle, starting at the depot such that the vehicle collects all the items from the customers and deposits them at the depot and the total cumulative cost is minimized. The total weight of the cargo is at most Q at any point in time. We allow the vehicle to offload the cargo at the depot an arbitrary number of times. The objects to be transported are indivisible. Let a be the cost of moving empty vehicle per unit distance and b be the cost of moving unit weight cargo per unit distance. The cumulative cost of moving a vehicle unit distance with cargo weight w is $a + bw$. The solution to Cu-VRP is a collection of k directed cycles $T = \{T_1, T_2, T_3, \ldots, T_k\}$. Each customer node is in exactly one of the cycles. The total weight of the objects picked in a cycle T_j is at most Q. Each cycle starts and ends at the depot node and visits at least one customer. Let l_i denote the distance traveled by the vehicle after picking the object at customer i and offloading it at the depot, in some cycle T_j. Let $|T_j|$ be the length of directed cycle T_j. The total cumulative cost $C(T)$ of the travel schedule given by solution T is

$$C(T) = a \sum_{j=1}^{k} |T_i| + b \cdot \sum_{i=1}^{n} w_i l_i.$$

The vehicle travels a total distance of $\sum_{j=1}^{k} |T_i|$. Each object i with weight w_i travels a distance of l_i in T. Ideally our objective is to find a solution T^* such that $C(T^*) \leq C(T)$ for all solutions T.

1.2. Related work

Column generation is a widely used technique first proposed by Ford and Fulkerson [23]. For applications of column generation to vehicle routing problems please see the papers by Feillet [20] and Taillard [46]. The column generation algorithms decompose the VRP problem into the master problem and the subproblem. The master problem starts with some initial set of columns. In each iteration, a new column with negative reduced cost is generated by solving a sub-problem. Column generation algorithm terminates when no new column can be generated. Typically for vehicle routing problems, the master problem is a set covering problem (SCP) or a set partitioning problem (SPP). The sub-problem usually is an elementary shortest path problem with resource constraint (ESPPRC) where the solution is a path with the most negative reduced cost. The elementary condition on the path ensures that nodes are not repeated in the path. ESPPRC problems are known to be NP-Hard [17]. Beasley and Christofides [6] were the first to study ESPPRC. The dynamic programming algorithm of Desrochers et al. [16] was extended by Feillet et al. [21] to solve ESPPRC directly. Chabrier [10] developed a similar algorithm for ESPPRC. The relaxed version of ESPPRC problem without the elementary condition is known as the shortest path problem with resource constraint (SPPRC) [32]. A pseudo-polynomial time algorithm for SPPRC is due to Desrochers et al. [16]. For a recent survey on exact approaches for SPPRC, see Pugliese and Guerriero [41].

The first branch and bound algorithm for capacitated VRP (CVRP) is due to Christofides et al. [12]. Branch and cut algorithms for CVRP were given by Laporte [37], and Fisher [22]. Branch-cut-price algorithms are due to Fukasawa et al. [25] and Baldacci et al. [4]. Jin et al. [33] gave a column generation algorithm for split delivery VRPs. Liberatore et al. [38] and Qureshi et al. [42] gave a column generation algorithm for VRPs with soft time windows. Recently Lysgaard and Wohlk [40] describe a branch-cut-price algorithm for cumulative capacitated VRPs. The exact algorithms can typically solve only small-sized instances in the order of 100 clients or so. Numerous approximation and heuristic algorithms have therefore been developed for CVRP.

Kara et al. [35,34] give polynomial size integer programming formulations for the collection and delivery cases of Cu-VRP. They also note that the cumulative VRPs generalize problems such as the minimum latency problem (MLP) [8,7] and the k-traveling repairmen problem (k-TRP) [18]. A special case of the cumulative VRP arises when only a single offload is allowed, the capacity of the vehicle is infinite, and the travel schedule is single TSP tour of graph G. Constant factor approximation algorithms have been given for this case by Blum et al. [8]. Gaur et al. [26] gave constant factor approximation algorithms for four variants of Cu-VRP. Fukasawa, He and Song [24] recently gave a branch cut and price algorithm for the cumulative vehicle routing problem. Xiao and Konak [48] gave an MILP formulation and algorithms to solve a similar VRP called the Green Vehicle routing problem. For a survey of green VRPs related to the one that we consider please see the recent review due to Demir et al. [15]. Recently, Gaur et al. [27] gave constant factor approximation algorithms for Cumulative VRPs with stochastic demands.

1.3. Overview

We show that a method based on rounding solutions to linear program performs well in practice. The solution to the linear program is computed using column generation, and the pricing problem is solved heuristically using dynamic programming. We establish the efficacy of the approach by simulations on several sets of VRP instances available from. We theoretically bound the integrality gap for the set cover formulation. To the best of our knowledge, this is the first such study for the cumulative vehicle routing problem. This is a revised and an expanded version of the results that appeared in CALDAM 2015 [28].