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On the extremal values of the eccentric distance sum of trees with a given domination number

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ABSTRACT

Let G be a simple connected graph. The eccentric distance sum (EDS) of G is defined as $\xi^d(G) = \sum_{v\in V}\varepsilon_G(v)D_G(v)$, where $\varepsilon_G(v)$ is the eccentricity of the vertex v and $D_G(v) = \sum_{u\in V} d_G(u, v)$ is the sum of all distances from the vertex v. In this paper, the extremal tree among n-vertex trees with domination number γ satisfying $4 \le \gamma < \lceil \frac{n}{3} \rceil$ having the maximal EDS is characterized. This proves Conjecture 4.2 posed in Miao et al. (2015).

1. Introduction

In this paper, all graphs $G = (V, E)$ are finite, simple and undirected. For $x \in V$, $N_G(x)$ (or $N(x)$) is the set of vertices adjacent to x, and the *degree* of x, denoted by $d_G(x)$ (or $d(x)$) is $|N_G(x)|$ (or $|N(x)|$). We call u a *leaf* if $d(u) = 1$. We use $\Delta(G)$ and $\delta(G)$(or Δ and δ for short) to denote the *maximum degree* and *minimum degree* of G, respectively. For $u, v \in V$, the *distance* $d_G(u, v)$ is defined as the length of a shortest path between u and v in G. $D_G(v)$ denotes the sum of all distances from v. The *eccentricity* $\varepsilon_G(v)$ of a vertex v is the maximum distance from v to any other vertex. The diam(G) of a graph G is the maximum eccentricity of any vertex in G. For $W \subseteq V$, $G \setminus W$ denotes the graph obtained from G by deleting the vertices in W together with their incident edges. If $W = \{w\}$, we just write $G \setminus w$ for $G - \{w\}$. For $E' \subseteq E$, $G \setminus E'$ denotes the graph obtained from G by deleting the edges in E'. If $E' = \{e\}$, we just write $G \setminus e$ for $G - \{e\}$. If $U \subseteq V$, then $G[U]$ denotes the graph on U whose edges are precisely the edges of G with both ends in U. Let P_n be a path on n vertices. We use $l(P)$ to denote the length of a path P. For a real number x, we use $\lfloor x \rfloor$ to denote the greatest integer no greater than x and use $\lceil x \rceil$ to denote the least integer no less than x.

A subset M of $E(G)$ is called a *matching* of G if no two edges are adjacent in G. The *matching number* of G, denoted by $\alpha'(G)$, is defined as the maximum cardinality of matching sets of G. A subset S of $V(G)$ is called an *independent set* of G if no two vertices from S are adjacent in G. The *independence number* of G, denoted by $\alpha(G)$, is defined as the maximum cardinality of independent sets of G. A subset S of V is called a *dominating set* of G if for every vertex $v \in V \setminus S$, there exists a vertex $u \in S$ such that v is adjacent to u. The *domination number* of G, denoted by $\gamma(G)$, is defined as the minimum cardinality of dominating sets of G. For a connected graph G of order n, Ore [12] obtained that $\gamma(G) \le \frac{n}{2}$. Some results on the domination number can be found in [1,13,16].

The graph invariant-eccentric distance sum (EDS) was introduced by Gupta, Singh and Madan [3], which was defined as

$$\xi^d(G) = \sum_{v\in V}\varepsilon_G(v)D_G(v).$$

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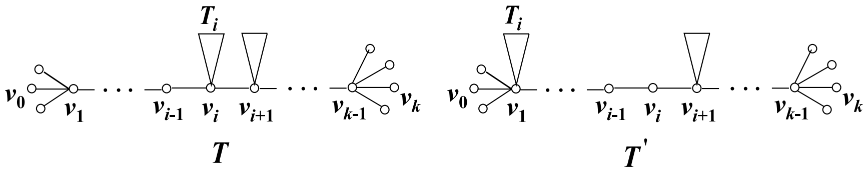

Fig. 2.1. An illustration of trees T and T' in A_2.

The eccentric distance sum can also be defined as

$$\xi^d(G) = \sum_{u,v\in V}(\varepsilon_G(u) + \varepsilon_G(v))d_G(u, v).$$

Many researchers have studied the eccentric distance sum of trees. Yu, Feng and Ilić [18] characterized the trees with the minimal EDS among the n-vertex trees of a given diameter. Li, Zhang, Yu, Feng [9] identified the trees with the minimal and second minimal eccentric distance sums among the n-vertex trees with matching number q and characterized the extremal trees with the second, third and fourth minimal eccentric distance sum among the n-vertex trees of a given diameter. Geng, Li and Zhang [2] and Miao [10] characterize the trees with the minimal EDS among n-vertex trees with domination number γ, and determine the trees with the maximal EDS among n-vertex trees with domination number γ satisfying $n = k\gamma$, where $k = 2, 3, \frac{n}{2}$ and $\frac{n}{3}$. They also identify the trees with the minimal and the maximal EDS among the n-vertex trees with k leaves. In [10] the trees having the maximal EDS among n-vertex trees with maximum degree Δ are characterized, the trees having the maximal or minimal EDS among n-vertex trees with independence number α, and the trees having the maximal EDS among n-vertex trees with matching number m are also determined, respectively. Other results about the EDS of graphs can also be found in [4,5,7,8]. For other eccentricity-based graph invariants one may refer to [6,11,14,15,17,19,20].

In this paper, we continue to study the eccentric distance sum of trees. The extremal tree among n-vertex trees with domination number γ satisfying $4 \le \gamma < \lceil \frac{n}{3} \rceil$ having the maximal EDS is characterized. This proves Conjecture 4.2 posed in [10].

2. Preliminaries

The following graph transformations A_1 and A_2 were posed in [5] and [2] respectively.

A_1: *Let T be a tree of order $n > 3$ and $e = uv$ be a nonpendant edge. Suppose that $T - e = T_1 \cup T_2$ with $u \in V(T_1)$ and $v \in V(T_2)$. A new tree T_0 is obtained by identifying the vertex u of T_1 with vertex v of T_2 and attaching a leaf to the $u(= v)$. T_0 is said to be obtained by running an edge-growing transformation of T (on edge $e = uv$), or e.g.t of T (on edge $e = uv$) for short.*

Let T be a tree and uv be a pendent edge with $d_T(v) = 1$ and $d_T(u) \ge 3$. Suppose $uw \in E(T)$ and $w \ne v$. Let $T_0 = T - \{uw\} + \{vw\}$. Then T_0 is said to be obtained by *running converse of e.g.t* of T on uw.

Lemma 2.1 ([5])**.** *Let T be a tree of order $n > 3$ and $e = uv$ be a nonpendant edge of T. If T_0 is a tree obtained from T by running one step of e.g.t (on edge $e = uv$), then we have $\xi^d(T_0) < \xi^d(T)$.*

A_2: *Suppose that $P = v_0v_1 \cdots v_i \cdots v_k$ is one of the longest paths contained in an n-vertex tree T with $d_T(v_1) \le d_T(v_{k-1})$ and $i = \min\{j | d_T(v_j) \ge 3, j = 2, 3, \ldots, k-2\}$. Let $T' = T - \{v_iu | u \in N_T(v_i) \setminus \{v_{i-1}, v_{i+1}\}\} + \{v_1u | u \in N_T(v_i) \setminus \{v_{i-1}, v_{i+1}\}\}$. Then T' is said to be obtained by running an A_2 transformation of T. See Fig. 2.1.*

Lemma 2.2 ([2])**.** *Let T be an n-vertex tree. If T' is a tree obtained from T by running an A_2, then we have $\xi^d(T') > \xi^d(T)$.*

Let $\mathcal{T}_{n,\gamma}$ be the set of all n-vertex trees with domination number γ. Let $P_l(a, b)$ be an n-vertex tree obtained by attaching a and b leaves to the two end vertices of $P_l = v_1v_2 \cdots v_l$, $(l \ge 2)$, respectively. Here, $a + b = n - l$, $a, b \ge 1$.

Lemma 2.3 ([2])**.** *Among all the trees from $\mathcal{T}_{n,\lceil \frac{n}{3} \rceil}$ with $n > 4$, the tree P_n has the maximal EDS.*

Lemma 2.4 ([2])**.** *Among $\mathcal{T}_{n,2}$ with $n \ge 4$, $P_4(\lfloor \frac{n-4}{2} \rfloor, \lceil \frac{n-4}{2} \rceil)$ maximizes the EDS.*

Lemma 2.5 ([10])**.** *Among $\mathcal{T}_{n,3}$ with $n \ge 10$, $P_7(\lfloor \frac{n-7}{2} \rfloor, \lceil \frac{n-7}{2} \rceil)$ maximizes the EDS.*

Lemma 2.6 ([2])**.** $\xi^d(P_l(1, n-l-1)) < \xi^d(P_l(2, n-l-2)) < \cdots < \xi^d(P_l(\lfloor \frac{n-l}{2} \rfloor, \lceil \frac{n-l}{2} \rceil))$.