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On the complexity of the identifiable subgraph problem, revisited

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ABSTRACT

A bipartite graph $G = (L, R; E)$ with at least one edge is said to be *identifiable* if for every vertex $v \in L$, the subgraph induced by its non-neighbors has a matching of cardinality $|L| - 1$. An ℓ-*subgraph* of G is an induced subgraph of G obtained by deleting from it some vertices in L together with all their neighbors. The Identifiable Subgraph problem is the problem of determining whether a given bipartite graph contains an identifiable ℓ-subgraph. The Min- and Max-Identifiable Subgraph problems are defined similarly, taking into account a given upper, resp. lower bound on the number of vertices from L in an identifiable ℓ-subgraph.

We show that the Identifiable Subgraph and Max-Identifiable Subgraph problems are polynomially solvable, thereby answering the question about the complexity of these two problems posed by Fritzilas et al. in 2013. We also complement a known APX-hardness result for the Min-Identifiable Subgraph problem by showing that two parameterized variants of the problem are W[1]-hard.

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1. Introduction

A *matching* in a graph is a subset of pairwise disjoint edges. A bipartite graph $G = (L, R; E)$ with at least one edge is said to be *identifiable* if for every vertex in L, the subgraph of G induced by its non-neighborhood has a matching of cardinality $|L| - 1$. Identifiable bipartite graphs were studied in several papers [4–7]; the property arises in the context of low-rank matrix factorization and has applications in data mining, signal processing, and computational biology. For further details on applications of notions and problems discussed in this paper, we refer to [4,5].

While the recognition problem for identifiable bipartite graphs is clearly polynomial using bipartite matching algorithms, several natural algorithmic problems concerning identifiable graphs turn out to be NP-complete [4,5]. In [4], three problems related to finding specific identifiable subgraphs were introduced. To state these problems, we need to recall the notion of an ℓ-subgraph of a bipartite graph (which appeared first in [4] and, in a slightly modified form, which we will adopt, in [7]). For a bipartite graph $G = (L, R; E)$ and vertex sets $X \subseteq L$, $Y \subseteq R$, we denote by $G[X, Y]$ the subgraph of G induced by $X \cup Y$.

Definition 1. Let $G = (L, R; E)$ be a bipartite graph. For a subset $J \subseteq L$, the ℓ-*subgraph of* G *induced by* J is the subgraph $G(J) = G[J, R \setminus N(L \setminus J)]$, where $N(L \setminus J)$ denotes the set of all vertices in R with a neighbor in $L \setminus J$. We say that a graph G' is an ℓ-*subgraph of* G if there exists a subset $J \subseteq L$ such that $G' = G(J)$.

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The following three problems are all related to finding identifiable ℓ-subgraphs of a given graph:

Identifiable Subgraph	
Instance:	A bipartite graph $G = (L, R; E)$.
Question:	Does G have an identifiable ℓ-subgraph?

Min-Identifiable Subgraph	
Instance:	A bipartite graph $G = (L, R; E)$ and an integer k.
Question:	Does G have an identifiable ℓ-subgraph induced by a set J with $\lvert J\rvert \leq k$?

Max-Identifiable Subgraph	
Instance:	A bipartite graph $G = (L, R; E)$ and an integer k.
Question:	Does G have an identifiable ℓ-subgraph induced by a set J with $\lvert J\rvert \geq k$?

In [4], the optimization version of the Min-Identifiable Subgraph problem was shown to be APX-hard. In the same paper it was shown that all three problems are polynomially solvable for trees, as well as for bipartite graphs $G = (L, R; E)$ such that the maximum degree of vertices in L is at most 2. In [7], restricted versions of the Identifiable Subgraph problem were studied, parameterizing the instances according to the maximum degree $\Delta(R)$ of vertices in R. Formally:

k-bounded Identifiable Subgraph	
Instance:	A bipartite graph $G = (L, R; E)$ with $\Delta(R) \leq k$.
Question:	Does G have an identifiable ℓ-subgraph?

It was shown in [7] that the k-bounded Identifiable Subgraph problem for $k \geq 3$ is as hard as Identifiable Subgraph problem in general and that the 2-bounded Identifiable Subgraph problem is solvable in linear time. The complexity of the Identifiable Subgraph and Max-Identifiable Subgraph problems in general bipartite graphs was left open by previous works [4,7].

In this paper, we establish the computational complexity of the Identifiable Subgraph and Max-Identifiable Subgraph problems, showing that both problems are solvable in polynomial time. The key idea to our approach is the observation that if the input graph $G = (L, R; E)$ is not identifiable, then one can compute in polynomial time a maximal subset $K \subseteq L$ no vertex of which is contained in any identifiable ℓ-subgraph of G. Such a set K is non-empty and can be safely deleted from the graph together with all its neighbors, thus reducing the problem to a smaller graph. If the algorithm finds an identifiable ℓ-subgraph of G, then it in fact finds an identifiable ℓ-subgraph of G induced by a largest possible subset of L, thereby also solving the Max-Identifiable Subgraph problem. The proof also shows that such a subgraph is unique.

In the second part of the paper, we complement the APX-hardness result for the optimization version of the Min-Identifiable Subgraph problem from [4] by studying the problem from the parameterized complexity point of view. We introduce two natural parameterized variants of the Min-Identifiable Subgraph problem and prove that both are W[1]-hard, by giving parameterized reductions from the well-known W[1]-hard Multicolored Clique (k) problem (cf. [1]).

The paper is structured as follows. In Section 2, we give the necessary definitions. In Section 3, we give a polynomial time algorithm that simultaneously solves the Identifiable Subgraph and Max-Identifiable Subgraph problems. In Section 4, we study the NP-hard Min-Identifiable Subgraph problem from the parameterized complexity point of view. Section 5 concludes the paper with some open questions.

2. Preliminaries

All graphs considered in this paper are finite, simple, and undirected. For a graph G, we denote by $V(G)$ the vertex set of G and by $E(G)$ its edge set. A *bipartite graph* is a graph G such that its vertex set can be partitioned into two sets L and R such that $L \cap R = \emptyset$ and $E(G) \subseteq \{\{\ell, r\}\,;\, \ell \in L \text{ and } r \in R\}$. In this paper, we will regard bipartite graphs as already *bipartitioned*, that is, given together with a fixed bipartition (L, R) of their vertex set, and hence use the notation $G = (L, R; E)$. For a graph G and a subset of vertices $X \subseteq V(G)$, we denote by $N_G(X)$ the neighborhood of X, i.e., the set of all vertices in $V \setminus X$ that have a neighbor in X. For a vertex $x \in V$, we write $N_G(x)$ for $N_G(\{x\})$, and denote the *degree* of x with $d_G(x) = |N_G(x)|$. In $N_G(X)$, $N_G(x)$, $d_G(x)$, we shall omit the subscript G if the graph is clear from the context. A *clique* in a graph is a set of pairwise adjacent vertices.

A *parameterized problem* is a language $Q \subseteq \Sigma^* \times \mathbb{N}$ where Σ is a finite alphabet; the second component, k, of instances $(x, k) \in \Sigma^* \times \mathbb{N}$ is called the *parameter*. A parameterized problem Q is *fixed-parameter tractable* (FPT) if there is a function $f : \mathbb{N} \to \mathbb{N}$, a constant c, and an algorithm A that decides $(x, k) \in Q$ in time $f(k)|x|^c$ for all $(x, k) \in \Sigma^* \times \mathbb{N}$. Let FPT denote the class of all fixed-parameter tractable parameterized problems. A *parameterized reduction* from $Q \subseteq \Sigma^* \times \mathbb{N}$ to $Q' \subseteq \Sigma'^* \times \mathbb{N}$ is a mapping $\pi : \Sigma^* \times \mathbb{N} \to \Sigma'^* \times \mathbb{N}$ such that there are functions $g, h : \mathbb{N} \to \mathbb{N}$ and a constant c with: $(x, k) \in Q$ if and only if $\pi((x, k)) \in Q'$, the parameter value k' of $(x', k') = \pi((x, k))$ is at most $g(k)$, and $\pi((x, k))$ can be computed in time $h(k)|x|^c$. It is well known that the existence of a parameterized reduction from Q to Q' and $Q' \in$ FPT implies that $Q \in$ FPT as well, and that parameterized reducibility is transitive. Accordingly, similarly to P vs. NP, there are hardness classes of problems that are suspected not to be FPT. In particular, it is believed that FPT $\neq$ W[1] and, under this assumption, a

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