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An Expanded Distributed Algorithm for Dynamic Resource Allocation over Strongly Connected Topologies

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Abstract—**This paper studies a distributed dynamic resource allocation problem for a multi-agent network with a strongly connected digraph, which has increased an equality constraint for individual node requirement from the point of dynamic prediction horizon. Each agent at each prediction horizon in the network is associated with a local resource and a quadratic convex cost function. The goal is to minimize the total cost in a distributed manner and subject to state constraints and collective equality constraints as well as an extra individual requirement simultaneously. With a fully distributed iteration setup, each node individual requirement is ensured and the globally optimal solution is obtained. The convergence of the proposed algorithm is verified theoretically according to eigenvalue perturbation theory and illustrated by simulation results.**

Keywords-dynamic resource allocation; multi-agent system; expanded distributed algorithm; optimal solution

I. INTRODUCTION

Recently, the resource allocation problem (RAP), researching how to allocate available resources to a number of nodes, has received increasing attentions due to its widely appears in many optimal decision tasks such as economic dispatch problem [1], optimal routing [2] and wireless sensor networks [3], [4].

At first, the size of network nodes is relatively small and many novel centralized methods have been proposed to solve the RAP [5]. For the general resource allocation problem, the global cost function is a sum of local convex functions [6], subject to some convex constraints such as equality constraints for the amount of resources and state inequality constraints to restrain the allocated resources. A number of recent literature have done breakthrough researches such as [7], [8] considered the equality constraint in decentralized way. To deal with the distributed RAP, [9] proposed the gradient descent method, [10] presented game theory and [11], [12] proposed consensus based algorithms under taking the state inequality constraint into consideration. Some of the recent literature on distributed optimization algorithm design includes distributed algorithms implemented both in discrete time [13] and continuous time [14].

To describe the problem as a RAP, graphs are adopted to model the communication topologies. [15] dealt with the equality constraints of distributed RAP under undirected graphs and directed switching topologies respectively, but they do not consider the state constraints. [16] have considered equality constraints and state constraints by directed switching topologies but the proposed algorithm can not realize in a distributed way.

But the literature mentioned above regard the RAP in a static way. All of them dealt with the resource allocation problem just based on the current state of the system, not from the perspective of dynamic moment in optimization problem. In this paper, we propose a distributed method to solve a dynamic resource allocation problem (DRAP), which has increased an equality constraint for individual node requirement at each prediction horizon.

To cope with the distributed RAP, the proposed expanded distributed algorithm is derived from [17] and [18]. In [17], a consensus based algorithm is proposed to solve economic dispatch problem in a distributed fashion, however, the algorithm only consider current horizon and the parameter can't be designed in a distributed way. In [18], a nonnegative-surplus based distributed algorithm is proposed for the average consensus problem which guarantees state averaging on arbitrary strongly connected digraphs. However, we consider directed and strongly connected topologies at every prediction horizon in this paper. In addition, the Lyapunov function based analysis developed in [18] took the nonnegative surplus as a premise, however, our algorithm can't ensure it due to another equality individual constraint. Therefore, the technical difficulty is that convergence analysis of the expanded distributed algorithm.

Notations: $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and integer numbers, respectively. $x_i^k(t)$ denotes the value of state variable of agent i at time k at iteration t. x denotes the state column stack vectors of $x_i^k(t)$. In the same way we use $\lambda, \lambda_i^k(t)$ and $s, s_i^k(t)$. Let $\mathbf{1} := [1, ..., 1]^T \in \mathbb{R}^{Nn}$ and $\bar{\mathbf{1}} := [1, ..., 1]^T \in \mathbb{R}^n$ be the column vector of all ones. $\mathbf{0} := [0, ..., 0]^T \in \mathbb{R}^{Nn}$ and $\bar{\mathbf{0}} := [0, ..., 0]^T \in \mathbb{R}^n$ be the column vector of all zeros. $\mathbf{0}_{N\times N}$ denotes the zero matrix of $N \times N$ dementions. Given a matrix M, the spectrum $\delta(M)$ is the set of its eigenvalues and the spectral radius $\rho(M)$ is the maximum modulus of its eigenvalues.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. *Preliminaries for Graphs*

For a network of n agents, we model their time-varying interconnection structure at time k by a dynamic digraph $\mathcal{G}(k)=(\mathcal{V},\mathcal{E}(k))$. Each node in $\mathcal{V}=\{1,...,n\}$ stands for an agent, and each directed edge (j,i) in $\mathcal{E}(k)\subseteq\mathcal{V}\times\mathcal{V}$ represents that agent j communicates to agent i at time k. For node $i,j\subseteq\mathcal{V}$, let $\mathcal{N}_i^-:=\{j\in\mathcal{V}:(j,i)\in\mathcal{E}(k)\}$ and $\mathcal{N}_i^+:=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}(k)\}$ denote the set of its $in-neighbors$ and $out-neighbors$. Note that at any time k of node i, in general $\mathcal{N}_i^+\neq\mathcal{N}_i^-$. In addition, $i\notin\mathcal{N}_i^+$ or $\mathcal{N}_i^-$. The in-degree and out-degree of node i are defined as $d_i^+(k):=\left|\mathcal{N}_i^+(k)\right|$ and $d_i^-(k):=\left|\mathcal{N}_i^-(k)\right|$, where $|\cdot|$ denotes the cardinality of a set.

Define two matrices $P,Q\in\mathbb{R}^{(Nn)\times(Nn)}$ associate with a strongly connected graph $\mathcal{G}(k)=(\mathcal{V},\mathcal{E}(k))$ as follows $\forall i,j\in\mathcal{V}$:

$$p_{ij}^k=\begin{cases}\frac{1}{d_i^++1} & \text{if } j\in\mathcal{N}_i^+\\ 0 & \text{otherwise}\end{cases},\ q_{ij}^k=\begin{cases}\frac{1}{d_i^-+1} & \text{if } i\in\mathcal{N}_j^-\\ 0 & \text{otherwise}\end{cases}$$

B. Distributed Resource Allocation Problem

Consider a network of n agents and every agent has N prediction horizon. Denote $x_i^k\in\mathbb{R}$ the amount of resources allocated to agent i at time k and suppose that x_i^k associate with a quadratic cost function:

$$F_i^k(x_i^k)=(x_i^k-\alpha_i^k)^2/2\beta_i^k+\gamma_i^k \tag{1}$$

where $\alpha_i^k,\gamma_i^k\in\mathbb{R}$, $\beta_i^k>0$. In many applications, each node i at time k may be allowed to have limited resources. Therefore, we consider an state inequality constraint for each x_i^k, i.e., $\underline{x}_i^k\leq x_i^k\leq\overline{x}_i^k$. For simplify, we assume $\overline{x}_i^k=\overline{x}_i$ and $\underline{x}_i^k=\underline{x}_i$ for all $k=0,1,...,N-1$. Due to consider each node has N prediction horizon, we increase an equality constraint for state and its sum of N prediction horizon satisfies resource requirement as desired. i.e., $\sum_{k=0}^{N-1}x_i^k=b_i$. The total amount of resources for allocation to n agents at time k is given by d^k. So, we consider the following DRAP:

$$\min_{x_1^0,...,x_n^{N-1}}\sum_{k=0}^{N-1}\sum_{i=1}^{n}F_i^k(x_i^k) \tag{2a}$$

subject to state inequality constraints

$$\underline{x}_i\leq x_i^k\leq\overline{x}_i \tag{2b}$$

and each node state equality requirement constraints

$$\sum_{k=0}^{N-1}x_i^k=b_i \tag{2c}$$

and total amount constraints for each prediction horizon

$$\sum_{i=1}^{n}x_i^k=d^k \tag{2d}$$

In order to make the dynamic resource allocation problem mentioned above solvable, define$b_i\in(N\underline{x}_i,N\overline{x}_i)$ and $d^k\in(n\underline{x}_i,n\overline{x}_i)$. To solve the DRAP proposed in (2) in a fully distributed manner, we put forward the following assumption for the solvability of the DRAP.

Asuumption 1: The digraph $\mathcal{G}(k)=(\mathcal{V},\mathcal{E}(k))$ for DRAP is strongly connected at prediction horizon of k.

Define the incremental cost for node i at time k is

$$J_i^k(x_i^k)=(dF_i^k(x_i^k))/dx_i^k=(x_i^k-\alpha_i^k)/\beta_i^k$$

Now, we present the optimality condition for the dynamic resource allocation problem (2), which can be simply obtained from the incremental cost criterion

$$\begin{cases}J_i^k((x_i^k)^*)=(\lambda_i^k)^*+\nu_i^* \text{ if } \underline{x}_i\leq(x_i^k)^*\leq\overline{x}_i\\ J_i^k((x_i^k)^*)\leq(\lambda_i^k)^*+\nu_i^* \text{ if } (x_i^k)^*=\overline{x}_i\\ J_i^k((x_i^k)^*)\geq(\lambda_i^k)^*+\nu_i^* \text{ if } (x_i^k)^*=\underline{x}_i\end{cases} \tag{3}$$

where $(x_i^k)^*$ is the optimal solution. $(\lambda_i^k)^*$, ν_i^* are the optimal incremental cost at time of k for n nodes and at N prediction horizon in a node, respectively.

III. Expanded Surplus Based Algorithm

In this section, we firstly ignore the state inequality constraints and develop a linear distributed algorithm 1 to solve the DRAP. We add in the state inequality constraints latter in algorithm 2.

According to the incremental cost criterion (3), when all nodes operate at the optimal configuration, incremental costs are equal to the optimal value, that is

$$\frac{(x_i^k)^*-\alpha_i^k}{\beta_i^k}=(\lambda_i^k)^*+\nu_i^* \tag{4}$$

Therefore, the optimal solution for each node can be calculated if the sum of optimal incremental cost $(\lambda_i^k)^*+\nu_i^*$ is known, i.e.,

$$(x_i^k)^*=\beta_i^k((\lambda_i^k)^*+\nu_i^*)+\alpha_i^k \tag{5}$$

In order to address the mismatch between $\sum_{i=1}^n x_i^k$ and d^k, we associate each node i at time k with s_i^k, which is called surplus. Define a projection function for $\lambda_i^k+\nu_i$

$$\phi(\lambda_i^k,\nu_i)=\beta_i^k(\lambda_i^k+\nu_i)+\alpha_i^k \tag{6}$$

The projection (6) is the inverse function of $J_i^k(\cdot)$ defined in (3) if and only if $\underline{x}_i\leq x_i^k\leq\overline{x}_i$. Now, we propose our expanded surplus based algorithm.

Algorithm 1 : Expanded Surplus Based Algorithm

Initialization :

$$\begin{cases}x_i^k(0)=\underline{x}_i,\\ \lambda_i^k(0)=J_i^k(x_i^k(0)),\\ s_1^k(0)=\cdots=s_{n-1}^k(0)=0, s_n^k(0)=d^k-\sum_{i=1}^n x_i^k(0)\end{cases} \tag{7}$$

Update :