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Uncertain linguistic fuzzy soft sets and their applications in group decision making

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ABSTRACT

The aim of this paper is to develop a novel concept of uncertain linguistic fuzzy soft sets (*ULFSSs*), which applies the notion of uncertain fuzzy set to the soft theory. The relationships between two *ULFSSs*, including the inclusion relation, the equal relation and the complement relation, are studied based on the binary relations. We also introduce some basic set operations for *ULFSSs*, such as the 'AND' and 'OR' operations, the algebra operations. The properties of these operations are also discussed. As an application of this new fuzzy soft set, we propose a *ULFSSs*-based group decision making model, in which the weights of decision makers are obtained from a non-linear optimization model according to the 'Technique for Order of Preference by Similarity to Ideal Solution' (TOPSIS) method and the maximum entropy theory. Finally, an assessment of sound quality problem is investigated to illustrate the feasibility and validity of the approach mentioned in this paper.

1. Introduction

Soft set theory, as a novel mathematical tool to handle vagueness and uncertainties, was introduced by Molodtsov [1] in 1999. Since the concept of soft set has been proposed, it was usually combined with other concepts, such as fuzzy set [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4] and type-2 fuzzy set [5]. The advantage of soft set is that it is free from the inadequacy of the parameterization tools, such as the probability theory, fuzzy sets and rough sets [1,4]. Recently, with these developments of soft set theory, kinds of applications have been studied, such as decision making [4–6,33–36], engineering [7], economics [8] and medical science [9], and so on. The meanings of kinds of soft sets are dealing with different practical issues based on vagueness and uncertainties.

Noting that in many complicated cases, because of lack of professional knowledge or data, time pressure and the decision maker's (DM) limited expertise related to the issues, the decision makers can only give uncertain linguistic information [10–12], which is a direct extension of linguistic information proposed by Zadeh [13]. The linguistic information or uncertain linguistic information has been studied and applied to many fields, including information retrieval [14], risk analysis [15], especially in decision making [16–19].

However, corresponding to the non-parametric mathematical tool, the situation that the approximate element of a soft set cannot be provided with objective evaluations has not been considered. The general framework of soft set is listed in Fig. 1.

According to Fig. 1, up until now, types of soft sets have been developed, but the soft set theory can also be enriched and improved by combining other types of information. Correspondingly, the approximate elements can be applied to different real situations.

In this paper, we will further extend a novel kind of soft set called uncertain linguistic fuzzy soft set to deal with the situation, which combines the concepts of uncertain linguistic fuzzy set and the soft set. Therefore, together with the initial soft set [1], kinds of fuzzy soft sets [2–5,20] and rough soft set [39], we have generalized the soft set theories so that types of information can be utilized. Besides, from the angle of linguistic term set, the proposed uncertain linguistic fuzzy set also provides a novel computation model of linguistic term set, which is shown to be a useful tool in group decision making.

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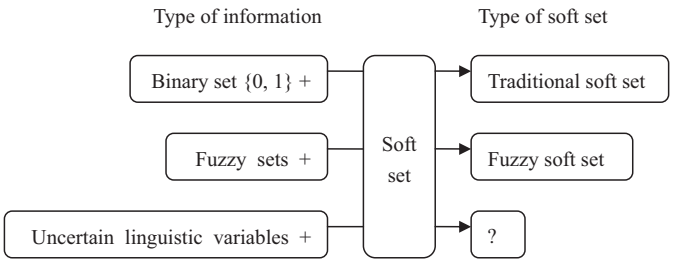

Fig. 1. The general framework of soft set.

To do this, the rest of this paper is structured as follows: In Section 2, we briefly review some basic definitions and notations for further consideration. Section 3 introduces the concept of uncertain linguistic fuzzy soft set (*ULFSS*) and studies the relationships between two *ULFSSs* based on binary relations. Besides, some set operations like 'AND' and 'OR' operations and algebra operations are also discussed in this section. We propose a new *ULFSSs*-based group decision making model in Section 4. An optimization model is developed to obtain the weighting vector of decision makers (DMs) based on 'Technique for Order of Preference by Similarity to Ideal Solution' (TOPSIS) approach and the maximum entropy theory. Finally, a case of assessment of sound quality problem is investigated in Section 5 to illustrate the feasibility and validity of the proposed approach. Some conclusions and possible further works are shown in Section 6.

2. Preliminaries

2.1. Fuzzy soft set

Assume that U is a real and finite universal set and A is a subset of the parameter set E

Definition 2.1. (see [1]). Let $P(U)$ be the power set of U, a pair (F, A) is said to be a soft set over U, where F is a mapping given by $F : A \to P(U)$.

It is worth noting that a soft set is not a set, but a parameterized family of subsets of U. For any $e \in A$, $F(e)$ is considered as the set of e-approximate elements of (F, A).

Definition 2.2. (see [22]). A fuzzy set X over U is defined by a mapping μ_X, where

$$\mu_X : U \to [0, 1], \tag{1}$$

μ_X is said to be the membership function of X, the element $\mu_X(u)$ is the membership of $u \in U$. X can be also represented in the following:

$$X = \{(\mu_X(u)/u) : u \in U,\ \mu_X(x) \in [0, 1]\}. \tag{2}$$

The set of all fuzzy sets over U can be denoted by $F(U)$.

In Ref. [21], Maji et al. introduced the definition of fuzzy soft set by combining Definitions 2.1 and 2.2, which can be shown as follows:

Definition 2.3. (see [21]). Suppose that $\tilde{P}(U)$ is the fuzzy power set of U, a pair $(\tilde{F}, A)$ is called a fuzzy soft set over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \to \tilde{P}(U)$.

According to Definition 2.3, the parameter sets are crisp in a fuzzy soft set, but the approximate functions are fuzzy subsets of U. Thus, the fuzzy soft set is more generalized than traditional soft set with crisp parameter sets and approximate functions. The following example will show the application of fuzzy soft set.

Example 2.1. The assessment of sound quality is of great importance for the judgment of sound, especially for musical works or sound equipments. Because of the blur of sound quality assessment, a multi-dimensional scaling method is usually applied to make an assessment according to the classification given by Linkwitz. lab. For instance, the clarity/articulation/speed, the spaciousness/openness, the instantaneous volume-range, the size of virtual sound image, the width/height of sound stage, etc.

Assume that $U = \{v_1, v_2, \ldots, v_6\}$ is a universal set and $E = \{e_1, e_2, \ldots, e_5\}$ is the factors that are considered by the classification given by Linkwitz. Lab. If $A = \{e_1, e_3, e_4, e_5\} \subseteq E$ and

$$\tilde{F}_A(e_1) = \left\{\frac{0.6}{v_1}, \frac{0.3}{v_2}, \frac{0.9}{v_3}, \frac{0.5}{v_4}, \frac{0.8}{v_5}, \frac{0.3}{v_6}\right\}$$

$$\tilde{F}_A(e_4) = \left\{\frac{0.3}{v_1}, \frac{0.9}{v_2}, \frac{0.6}{v_3}, \frac{0.9}{v_4}, \frac{0.3}{v_5}, \frac{0.8}{v_6}\right\},$$

$$\tilde{F}_A(e_5) = \left\{\frac{0.3}{v_1}, \frac{0.6}{v_2}, \frac{0.4}{v_3}, \frac{0.6}{v_4}, \frac{0.9}{v_5}, \frac{0.7}{v_6}\right\}.$$

The tabular form of such fuzzy soft set is represented in Table 1.