ARTICLE IN PRESS

Discrete Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

The pessimistic diagnosability of three kinds of graphs

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ARTICLE INFO

Article history:
Received 7 April 2016
Received in revised form 20 July 2016
Accepted 24 September 2016
Available online xxxx

Keywords:
Interconnection network
Pessimistic diagnosability
t/t-diagnosable
PMC model

ABSTRACT

A system is t/t-diagnosable if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free processor mistaken as a faulty one. The pessimistic diagnosability of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable. The pessimistic diagnosability of alternating group graphs AG_n (Tsai, 2015); BC networks (Fan, 2005; Tsai, 2013); the k-ary n-cube networks Q_n^k, (Wang et al., 2012); regular graphs including the alternating group networks AN_n (Hao et al., 2016) etc. But most of these results are about networks G with $cn(G) \leq 2$ (where $cn(G)$ is the maximum number of common neighbors for any two distinct vertices). In this paper, we study the pessimistic diagnosability of three kinds of graphs which are (n, k)-arrangement graphs $A_{n,k}$, (n, k)-star graphs $S_{n,k}$ and balanced hypercubes BH_n, where $cn(A_{n,k}) = cn(S_{n,k}) = n - k - 1$ and $cn(BH_n) = 2n$. We proved that $t_p(A_{n,k}) = (2k - 1)(n - k) - 1$ for $n \geq k + 2$ and $k \geq 3$, $t_p(S_{n,k}) = n + k - 3$ for $n \geq k + 2$ and $k \geq 3$, and $t_p(BH_n) = 2n$ for $n \geq 2$. As corollaries, the pessimistic diagnosability of the known results about AG_n and AN_n is obtained directly.

1. Introduction

An interconnection network can be modeled as an undirected graph with each processor represented as a vertex and each communication link represented as an undirected edge. The study of interconnection networks has been an important research area for parallel and distributed computer systems. Network reliability is one of the major factors in designing the topology of an interconnection network.

With the rapid development of technology, a multiprocessor system may contain thousands of processors. As a significant increase in the number of processors, fault diagnosis of interconnection networks has become increasingly important. For the purpose of self-diagnosis of a system, a number of models have been proposed. In 1967, Preparata et al. [21] established a foundation of system diagnosis and an original diagnostic model, called the *PMC model* (i.e., Preparata, Metze and Chien's model). In the PMC model, every processor performs tests on its neighbors by communication links between them. When one processor tests another, the tester declares the tested processor to be fault-free or faulty depending on the test result; the result is always accurate if the tester is fault-free, but if the tester is faulty, the result is unreliable. The collection of all test outputs is called the syndrome σ. For a given syndrome σ, a subset of vertices $F \subseteq V$ is said to be consistent with σ if σ can arise from the circumstance that all vertices in F are faulty and all vertices in $V - F$ are fault free.

A system is said to be *t-diagnosable* if all faulty units can be identified provided the number of faulty units present does not exceed t. The diagnosability of a system is the maximal number of faulty processors that the system can guarantee to diagnose. The diagnosability of many networks has been investigated in the literature, for example, see

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http://dx.doi.org/10.1016/j.dam.2016.09.033
0166-218X/

Please cite this article in press as: M.-M. Gu, R.-X. Hao, The pessimistic diagnosability of three kinds of graphs, Discrete Applied Mathematics (2016), http://dx.doi.org/10.1016/j.dam.2016.09.033

[3,2,11,13,16,18–20,32] etc. Under the precise strategy, a fault-set F identified only contains all faulty vertices. Nevertheless, using the pessimistic strategy, a fault-set F identified is allowed to contain all faulty vertices and some fault-free vertices. The pessimistic diagnosis strategy proposed by Kavianpour and Friedman [17] is a classic strategy based on the PMC model. Then Chwa and Hakimi [9] showed that there exists at most one fault-free vertex which is contained in F under this diagnosis strategy. In this strategy, all faulty processors can be isolated within a set having at most one fault-free processor.

Definition 1. A system is *t/t-diagnosable* if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free processor mistaken as a faulty one. The *pessimistic diagnosability* of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable.

Using the PMC model with a pessimistic strategy, the t/t-diagnosable of many interconnection networks has been received much attention. For example, see [1,4,11,12,24,25,28,31] etc. The pessimistic diagnosability of alternating group graphs AG_n and the hypercube-like networks (BC graphs) was obtained by Tsai in [23,22], respectively. Hao et al. [14] gave the pessimistic diagnosabilities of some general regular graphs and obtained the pessimistic diagnosability of alternating group networks AN_n. But almost all of these known results are about the network G with $cn(G) \leq 2$, where $cn(G)$ is the maximum number of common neighbors for any two distinct vertices. Motivated by this, in this paper, we consider the pessimistic diagnosabilities of three kinds of graphs: (n, k)-arrangement graphs $A_{n,k}$ with $cn(A_{n,k}) = n-k-1$ $(n \geq k+2)$, (n, k)-star graphs $S_{n,k}$ with $cn(S_{n,k}) = n-k-1$ $(n \geq k+2)$ and balanced hypercubes BH_n with $cn(BH_n) = 2n$ $(n \geq 2)$.

By exploring the topology structures of these kinds of graphs, we prove that $t_p(A_{n,k}) = (2k-1)(n-k)-1$ for $n \geq k+2$ and $k \geq 3$, $t_p(S_{n,k}) = n+k-3$ for $n \geq k+2$ and $k \geq 3$, and $t_p(BH_n) = 2n$ for $n \geq 2$. As corollaries, the pessimistic diagnosability of the known results $t_p(AG_n) = 4n-11$ for the alternating group graphs AG_n and $t_p(AN_n) = 2n-5$ for the alternating group networks AN_n is obtained directly.

The remainder of this paper is organized as follows. Section 2 introduces preliminaries and the structure properties of (n, k)-arrangement graph $A_{n,k}$, (n, k)-star graph $S_{n,k}$ and balanced hypercube BH_n. Section 3 concentrates on the pessimistic diagnosabilities of three kinds of graphs. Section 4 concludes the paper.

2. Preliminaries

In this section, we give some terminologies and notations of combinatorial network theory. For notation and terminology not defined here we follow [27].

We use a graph, denoted by $G = (V(G), E(G))$, to represent an interconnection network, where a vertex $u \in V(G)$ represents a processor and an edge $(u, v) \in E(G)$ represents a link between vertices u and v. Two vertices u and v are *adjacent* if $(u, v) \in E(G)$, the vertex u is called a neighbor of v, and vice versa. For a vertex $u \in V(G)$, let $N_G(u)$ denote a set of vertices in G adjacent to u. For a vertex set $U \subseteq V(G)$, the *neighborhood* of U in G is defined as $N_G(U) = \bigcup_{v\in U} N_G(v) - U$. If $|N_G(u)| = k$ for any vertex in G, then G is *k-regular*. Let G be a connected graph, if $G - S$ is still connected for any $S \subseteq V(G)$ with $|S| \leq k-1$, then G is *k-connected.*

For any two vertices u and v in G, let $cn(G; u, v)$ denote the number of vertices who are the neighbors of both u and v, that is, $cn(G; u, v) = |N_G(u) \cap N_G(v)|$. Let $cn(G) = \max\{cn(G; u, v) : u, v \in V(G)\}$.

A graph H is a *subgraph* of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. The *components* of a graph G are its maximally connected subgraphs. A component is *trivial* if it has only one vertex; otherwise, it is *nontrivial*. The *connectivity* of a graph G, denoted by $\kappa(G)$, defined as the minimum number of vertices whose removal results in a disconnected or trivial graph.

2.1. The arrangement graph $A_{n,k}$ and its properties

The (n, k)-*arrangement graph*, denoted by $A_{n,k}$, was proposed by Day and Tripathi [10] in 1992. The definition of $A_{n,k}$ is as follows.

Definition 2. Given two positive integers n and k with $n > k$, let $[n]$ denote the set $\{1, 2, \ldots, n\}$, and let $P_{n,k}$ be a set of arrangements of k elements in $[n]$. The (n, k)-arrangement graph, denoted by $A_{n,k}$, has vertex-set $P_{n,k}$ and two vertices are adjacent if and only if they differ in exactly one position.

$A_{n,k}$ is $k(n-k)$-regular, $k(n-k)$-connected with $\frac{n!}{(n-k)!}$ vertices, vertex-transitive and edge-transitive (see [10]). Clearly, $A_{n,1}$ is isomorphic to the complete graph K_n and $A_{n,n-1}$ is isomorphic to the n-dimensional star graph S_n. Chiang and Chen [8] showed that $A_{n,n-2}$ is isomorphic to the n-alternating group graph AG_n. Since $A_{n,n-1} \cong S_n$, which has been discussed in [31], to avoid duplication of discussion, we may assume $n \geqslant k+2$ and $k \geqslant 2$ in the following discussion.

For two distinct i and j in $[n]$, let $V_{n,k}^{j:i}$ be the set of all vertices in $A_{n,k}$ with the jth position being i, that is, $V_{n,k}^{j:i} = \{p | p = p_1p_2 \cdots p_j \cdots p_k \in P_{n,k}, p_j = i\}$. For a fixed position $j \in [n]$, $\{V_{n,k}^{j:i} | 1 \leq i \leq n\}$ forms a partition of $V(A_{n,k})$. Let $A_{n,k}^{j:i}$ denote the subgraph of $A_{n,k}$ induced by $V_{n,k}^{j:i}$. Then for each $j \in [n]$, $A_{n,k}^{j:i}$ is isomorphic to $A_{n-1,k-1}$. Thus, $A_{n,k}$ can be recursively constructed from n copies of $A_{n-1,k-1}$. It is easy to check that each $A_{n,k}^{j:i}$ is a subgraph of $A_{n,k}$, and we say that $A_{n,k}$ is decomposed into n

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