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The 2-good-neighbor connectivity and 2-good-neighbor diagnosability of bubble-sort star graph networks☆

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ABSTRACT

Connectivity plays an important role in measuring the fault tolerance of interconnection networks. The g-good-neighbor connectivity of an interconnection network G is the minimum cardinality of g-good-neighbor cuts. Diagnosability of a multiprocessor system is one important study topic. A new measure for fault diagnosis of the system restrains that every fault-free node has at least g fault-free neighbor vertices, which is called the g-good-neighbor diagnosability of the system. As a famous topology structure of interconnection networks, the n-dimensional bubble-sort star graph BS_n has many good properties. In this paper, we prove that 2-good-neighbor connectivity of BS_n is $8n-22$ for $n \geq 5$ and the 2-good-neighbor connectivity of BS_4 is 8; the 2-good-neighbor diagnosability of BS_n is $8n-19$ under the PMC model and MM* model for $n \geq 5$.

1. Introduction

Many multiprocessor systems take interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph where nodes represent processors and links represent communication links between processors. We use graphs and networks interchangeably. For the system, study on the topological properties of its network is important. Furthermore, some processors may fail in the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of a system G is the maximum value of t such that G is t-diagnosable [8,10,17]. For a t-diagnosable system, Dahbura and Masson [8] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

Several diagnosis models (e.g., Preparata, Metze, and Chien's (PMC) model [28], Barsi, Grandoni, and Maestrini's (BGM)model [2], and Maeng and Malek's (MM) model [26]) have been proposed to investigate the diagnosability of multiprocessor systems. In particular, two of the proposed models, the PMC model and MM model, are well known and widely used. In the PMC model, the diagnosis of the system is achieved through two linked processors testing each other. In the MM model, to diagnose a system, a node sends the same task to two of its neighbor vertices, and then compares their

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responses. For this reason, the MM model is also said to be the comparison model. Sengupta and Dahbura [8] proposed a special case of the MM model, called the MM^* model, in which each node must test its any pair of adjacent nodes. Numerous studies have been investigated under the PMC model and MM model or MM^* model, see [7,10,17,22,27,40].

In the traditional measurement of a system-level diagnosability for the multiprocessor system, one generally assumes that any subset of processors may simultaneously fail. If all the neighbor vertices of some node v are faulty simultaneously, it is impossible to determine whether v is faulty or fault-free. As a consequence, the diagnosability of a system is less than its minimum node degree. However, in a large-scale multiprocessor system, we can safely assume that all neighbor vertices of any node do not fail at the same time. Based on this assumption, Lai et al. [17] introduced the restricted diagnosability of the multiprocessor system called the conditional diagnosability of the system. They consider the situation that any fault set cannot contain all the neighbor vertices of any vertex in a system. The restricted diagnosability of the system has received much attention [5,6,23–25,38]. In 2012, Peng et al. [27] proposed a measure for fault diagnosis of the system, namely, the g-good-neighbor diagnosability (which is also called the g-good-neighbor conditional diagnosability), which requires that every fault-free node has at least g fault-free neighbors. In [27], they studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the PMC model. In [34], Wang and Han studied the g-good-neighbor diagnosability of the n-dimensional hypercube under MM^* model. Yuan et al. [40,41] studied that the g-good-neighbor diagnosability of the k-ary n-cube ($k \geq 3$) under the PMC model and MM^* model. As a favorable topology structure of interconnection networks, the Cayley graph $C\Gamma_n$ generated by the transposition tree Γ_n has many good properties. In [33,35], Wang et al. studied the g-good-neighbor diagnosability of $C\Gamma_n$ under the PMC model and MM^* model for $g = 1, 2$. In 2015, Zhang et al. [42] proposed a new measure for fault diagnosis of the system, namely, the g-extra diagnosability, which restrains that every fault-free component has at least $(g + 1)$ fault-free nodes. In [42], they studied the g-extra diagnosability of the n-dimensional hypercube under the PMC model and MM^* model.

The star graph and the bubble-sort graph have been proved to be an important viable candidate for interconnecting a multiprocessor system [1]. The feature of the star graph include low degree of nodes, small diameter, symmetry, and high degree of fault-tolerance. For details, see [7,9,12–14,18–21,29–32,39]. The diagnosabilities of the star graph under the PMC model and MM model were studied in [16,43]. Lin et al. [22] showed that the conditional diagnosability of the star graph S_n under the comparison diagnosis model is $3n - 7$. Guo et al. [11] showed that the conditional diagnosability of the bubble-sort star graph BS_n under the MM model is $6n - 15$ for $n \geq 6$ and under the PMC model is $8n - 21$ for $n \geq 5$. In 2016, Wang et al. [36] studied that the 2-extra diagnosability of BS_n under the PMC model and MM^* model. In this paper, the diagnosability of the n-dimensional bubble-sort star graph BS_n under the PMC model and MM^* model has been studied. It is proved that (a) the 2-good-neighbor connectivity of BS_n is $8n - 22$ for $n \geq 5$ and the 2-good-neighbor connectivity of BS_4 is 8; (b) the 2-good-neighbor diagnosability of BS_n is $8n - 19$ under the PMC model and MM^* model for $n \geq 5$; (c) the 3-extra diagnosability of BS_n is less than or equal to $8n - 19$ under the *PMC* model and MM^* model.

The rest of this paper is organized as follows: In Section 2, we provide the terminology and preliminaries for the system diagnosis. In Section 3, we shall show the 2-good-neighbor connectivity of BS_n is $8n - 22$ and the 2-good-neighbor connectivity of BS_4 is 8. In Sections 4 and 5, we evaluate the 2-good-neighbor diagnosability of BS_n under the PMC model and MM^* model, respectively. Finally, the conclusion is given in Section 6.

2. Preliminaries

In this section, some definitions and notations needed for our discussion, the bubble-sort star graph, the PMC model and MM^* model are introduced.

2.1. Definitions and notations

A multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Given a nonempty vertex subset V' of V, the induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v is the number of edges incident with v. We denote by $\delta(G)$ the minimum degrees of vertices of G. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor vertex of v for $u \in N_G(v)$. Let $S \subseteq V$. We use $N_G(S)$ to denote the set $\cup_{v\in S} N_G(v) \setminus S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if for any vertex v, $d_G(v) = k$. A graph is bipartite if its vertex set can be partitioned into two subsets X and Y so that every edge has one end in X and one end in Y; such a partition (X, Y) is called a bipartition of the graph, and X and Y its parts. We denote a bipartite graph G with bipartition (X, Y) by $G = (X, Y; E)$. If $G = (X, Y; E)$ is simple and every vertex in X is joined to every vertex in Y, then $G = (X, Y; E)$ is called a complete bipartite graph, denoted by $K_{n,m}$, where $|X| = n$ and $|Y| = m$. Let $G = (V, E)$ be a connected graph. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. A fault set $F \subseteq V$ is called a g-good-neighbor faulty set if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A g-good-neighbor cut of a graph G is a g-good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is said to be the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. Let $B_1, \ldots, B_k (k \geq 2)$ be the components of $G - F$. If $|V(B_1)| \leq \cdots \leq |V(B_k)| (k \geq 2)$, then B_k is called the maximum component of $G - F$. For graph-theoretical terminology and notation not defined here we follow [3].

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