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Note

A note on an induced subgraph characterization of domination perfect graphs

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ABSTRACT

Let $\gamma(G)$ and $\iota(G)$ be the domination and independent domination numbers of a graph G, respectively. Introduced by Sumner and Moore (1979), a graph G is domination perfect if $\gamma(H) = \iota(H)$ for every induced subgraph $H \subseteq G$. In 1991, Zverovich and Zverovich proposed a characterization of domination perfect graphs in terms of forbidden induced subgraphs. Fulman (1993) noticed that this characterization is not correct. Later, Zverovich and Zverovich (1995) offered such a second characterization with 17 forbidden induced subgraphs. However, the latter still needs to be adjusted.

In this paper, we point out a counterexample. We then give a new characterization of domination perfect graphs in terms of only 8 forbidden induced subgraphs and a short proof thereof. Moreover, in the class of domination perfect graphs, we propose a polynomial-time algorithm computing, given a dominating set D, an independent dominating set Y such that $|Y| \leq |D|$.

1. Introduction

1.1. Basic definitions and notations

In this paper, graphs are undirected and simple. Standard notions are explained, for instance, by Diestel [11]. V and E denote the vertex and edge sets of a graph G, respectively. For a given vertex v, $N(v)$ denotes the set of all neighbors (i.e. adjacent vertices) while, for a given vertex set X, $G[X]$ denotes the subgraph of G induced by X. Moreover, if G and H are two graphs, we say that G is H-free if H does not appear as an induced subgraph of G. Furthermore, if G is H_1-free, H_2-free, …, H_k-free for some graphs $H_1, H_2, \ldots, H_k$, we say that G is $(H_i)_{i=1}^k$-free.

A *dominating set* of a graph $G = (V, E)$ is a set D of vertices such that every vertex $v \in V \setminus D$ has at least one neighbor in D. The *domination number* of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set. A dominating set with such cardinality is called *minimum* while a dominating set is *minimal* if no proper subset is a dominating set.

A graph is *complete* if it contains all possible edges. A set D of vertices is *independent* (also called *stable*) if the subgraph induced by D has no edge. An independent set X of a graph $G = (V, E)$ is *maximal* if for every vertex $v \in V \setminus X$, $X \cup \{v\}$ is not independent. A dominating set D of a graph G is called *independent* if D is independent. It is known [4,6], that an independent dominating set is a maximal independent set, and conversely. The *independent domination number* of a graph G, denoted by $\iota(G)$, is the minimum cardinality of an independent dominating set in G. Thus, an independent dominating set is *minimum* if its cardinality is minimum.

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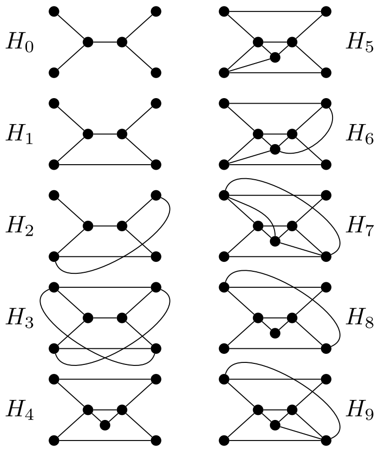

Fig. 1. An illustration of graphs H_i, for $i = 0, \ldots, 9$.

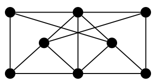

Fig. 2. An illustration of the graph H_{10}.

Sumner and Moore [23] introduced the notion of *domination perfect graph*, as a graph G such that $\gamma(H) = \iota(H)$, for all induced subgraphs H of G. A graph is said *minimal domination perfect* if the graph is not domination perfect but all proper induced subgraphs are.

1.2. Previous works

The class of domination perfect graphs has been studied. Looking for a characterization, many authors focused on special subclasses of graphs. We present here a brief survey on domination perfect graphs.

The line graph $L(T)$ of a tree T is always domination perfect [7,20]. More generally, every line graph is domination perfect, proved by Allan and Laskar [1] and independently by Gupta (see Theorem 10.5 [17]). In fact, Allan and Laskar gave a sufficient condition in the following theorem.

Theorem 1 (*Allan and Laskar [1]*)**.** *Every claw-free graph is domination perfect.*

Topp and Volkmann [24] generalized their results to new classes of graphs.

Theorem 2 (*Topp and Volkmann [24]*)**.** *If G is H_{10}-free and $(G_i)_{i=1}^{13}$-free (see Figs. 2 and 3) then G is domination perfect.*

As observed in [27], the original version of this theorem in [24] was stated with two additional graphs, which were shown to be redundant.

Harary and Livingston [18] studied the class of domination perfect trees and offered a complex characterization of this class. Other characterizations of these particular trees are mentioned in [9,13,19]. Actually, determining a minimum dominating set and a minimum independent dominating set in trees can be achieved in linear time [7,10,14].

Sumner [22] gave a characterization of domination perfect graphs in the classes of chordal and planar graphs while Zverovich and Zverovich [26] tackled the case of triangle-free graphs. Consider the class $\mathcal{S}$ of graphs defined by

$$\mathcal{S} = \{H \text{ graph on at most 8 vertices} \mid \gamma(H) = 2, \iota(H) > 2\}.$$

Theorem 3.

- *(Sumner [22]) Let G be a chordal graph. G is domination perfect if and only if G is H_0-free.*
- *(Sumner [22]) Let G be a planar graph. G is domination perfect if and only if G is $\mathcal{S}$-free.*
- *(Zverovich and Zverovich [26]) Let G be a triangle-free graph. G is domination perfect if and only if G is $(H_i)_{i=0}^{3}$-free.*

where graphs H_i are drawn in Fig. 1.

Sumner and Moore [23] attempted to extend previous results to all graphs.

Theorem 4 (*Sumner and Moore [23]*)**.** *If G is $\mathcal{S}$-free and G is H_{10}-free then G is domination perfect, where H_{10} is depicted in Fig. 2.*