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Counting independent sets in tree convex bipartite graphs

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ABSTRACT

The problems of counting independent sets, maximal independent sets, and independent perfect dominating sets are #P-complete for bipartite graphs, but can be solved in polynomial time for convex bipartite graphs, which are a subclass of bipartite graphs This paper studies these problems for tree convex bipartite graphs, which are a class of graphs between bipartite graphs and convex bipartite graphs. A bipartite graph G with bipartition $(X\ Y)$ is called tree convex, if a tree T defined on X exists, such that for every vertex y in Y, the neighbors of y form a subtree of T If the associated tree T is simply a path, then G is just a convex bipartite graph. This paper first proves that the problems of counting independent sets, maximal independent sets, and independent perfect dominating sets remain #P-complete for tree convex bipartite graphs even when the associated tree T is only a comb or a star. This paper then presents polynomial-time algorithms to solve these problems for tree convex bipartite graphs when the associated tree T is restricted to a triad, which consists of three paths with one common endpoint.

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1. Introduction

Let $G = (V, E)$ be a graph with set of vertices V and set of edges E. An *independent set* (IS) in G is a subset D of V such that no two vertices of D are mutually adjacent. The *maximal independent set* (MIS) is an IS that is not a subset of any other IS. A *dominating set* in G is a subset D of V such that every vertex that is not in D is adjacent to at least one vertex in D. An *independent dominating set* in G is a set of vertices of G that is both independent and dominating in G. Since every dominating set that is independent must be maximal independent, independent dominating sets are identically MISs. An independent dominating set D is an *independent perfect dominating set* (IPDS) if every vertex that is not in D is adjacent to exactly one vertex in D. Let $IS(G)$, $MIS(G)$, and $IPDS(G)$ be the sets of all ISs, MISs, and IPDSs in G, respectively. Then, the inclusions $IPDS(G) \subseteq MIS(G) \subseteq IS(G)$ hold by definitions.

This paper investigates the problems of computing the numbers of ISs, MISs, and IPDSs in a graph, denoted as #IS, #MIS, and #IPDS, respectively. Provan and Ball [10] verified that the problem #IS is #P-complete for general graphs and even for bipartite graphs. Valiant [13] defined the class of #P problems as those that involve counting access computations for problems in NP, and the class of #P-complete problems includes the hardest problems in #P. As is well known, all exact algorithms for solving #P-complete problems have exponential time complexity, so efficient exact algorithms for solving this class of problems are unlikely to exist. However, this complexity can be reduced by considering only a restricted subclass of #P-complete problems.

The complexities of the problems #IS, #MIS, and #IPDS have been extensively studied on various graphs such as bipartite graphs [10], interval graphs [7], chordal graphs [9], and tolerance graphs [8], and these complexities are presented in Fig. 1.

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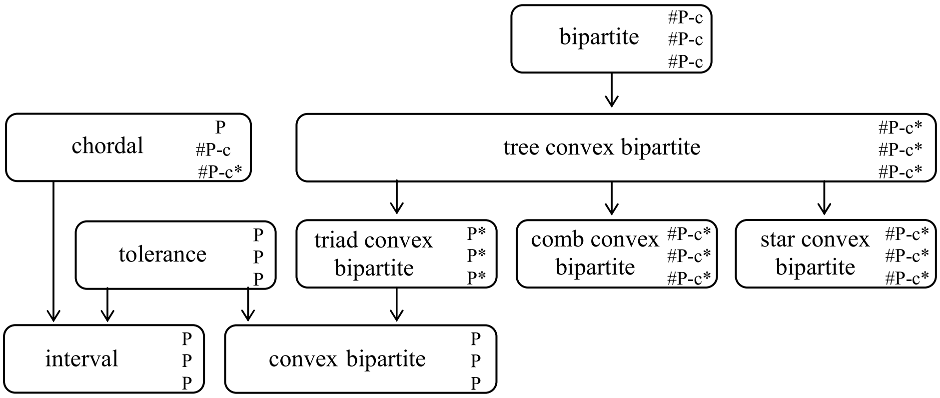

Fig. 1. Inclusion relations between some graph classes. A → B means that class A contains class B. Label at right of box that corresponds to each class indicates complexity of problems #IS (top), #MIS (middle), and #IPDS (bottom). #P-c and P denote #P-complete and polynomial-time, respectively. Symbol * indicates a main contribution of this paper.

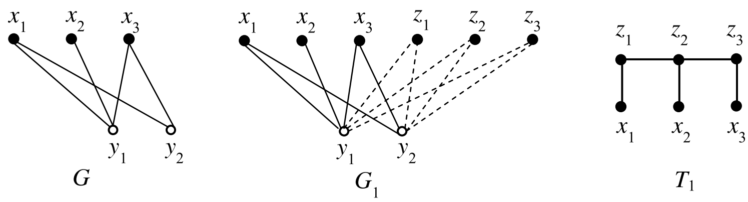

Fig. 2. Example of Constructions 1 and 2.

This paper concerns the class of *tree convex bipartite* graphs [5,11,6,2], which lies between bipartite graphs and convex bipartite graphs [3]. A bipartite graph $G = (X, Y, E)$ is called *tree convex* if there exists a tree $T = (X, F)$ such that, for each y in Y, the neighbors of y induce a subtree in T. When T is a star or a comb, G is called *star convex bipartite* or *comb convex bipartite*, respectively. A *comb graph* is a graph that is obtained by attaching a pendant leaf (tooth) to each vertex of a path (backbone). When T is a *triad tree*, which consists of three paths with a common end, G is called *triad convex bipartite*. When T is a path, G is called path convex bipartite or just *convex bipartite*.

This paper shows that the problems #IS, #MIS, and #IPDS are #P-complete even for star convex bipartite graphs and comb convex bipartite graphs, but all of these problems can be solved in polynomial time for triad convex bipartite graphs with given a triad tree.

2. Hardness results

This section proves that the problems #IS, #MIS, and #IPDS for comb convex bipartite graphs and star convex bipartite graphs are #P-complete, immediately implying that all such problems for tree convex bipartite graphs are #P-complete. Before proceeding, some notation must be introduced. For a graph $G = (V, E)$, let $N_G(v) = \{u \in V | (u, v) \in E\}$ represent the neighborhood of a vertex v in G and $N_G[v] = \{v\} \cup N_G(v)$ represent the closed neighborhood of a vertex v in G. For $v \in V$ and $V^* \subseteq V$, let $G - v$ represent the subgraph of G that is induced by the vertices of $V \setminus \{v\}$ and let $G - V^*$ represent the subgraph of G that is induced by the vertices of $V \setminus V^*$.

2.1. Comb convex bipartite graphs

Theorem 1. *The problem #IS for comb convex bipartite graphs is #P-complete.*

Proof. The reduction is performed from the problem of counting the ISs in a bipartite graph, which has been proven to be #P-complete in [10].

Construction 1. Let $G = (X, Y, E)$ be a bipartite graph with $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_m\}$. A bipartite graph $G_1 = (X \cup Z, Y, E \cup E')$ is constructed from G, with $Z = \{z_1, z_2, \ldots, z_n\}$ and $E' = \{(z, y) | z \in Z, y \in Y\}$. This construction is similar to the so-called *canonical transformation* [2]. Fig. 2 shows an example of Construction 1.

The proof of Theorem 1 can be split into proofs of Claims 1.1 and 1.2.

Claim 1.1. *G_1 is a comb convex bipartite graph.*

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