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Application of neighborhood sequences in communication of hexagonal networks

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This paper is dedicated to the memory of my Ph.D. supervisor, Prof. Mátyás Arató (1931–2015)

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ABSTRACT

In this paper communication in hexagonal network is analyzed. Every vertex of the network has direct connections to its neighbors. They communicate with neighborhood sequences varying the used neighborhood relations in each step. Theoretical results, such as algorithm to provide a shortest path and properties of *B*-distances are shown. Applications for broadcasting information and for two-way communications are presented and discussed.

1. Introduction

The communication is one of the major application fields of Information Technology and Computer Science. In practice, the communication in networks can be found in networks of people, in networks of processors, in computer networks and also in mobile networks [20]. Two basic types of networks are used: the broadcast and the point-to-point networks. Broadcast networks have a single communication channel that is shared by all machines. In contrast, point-to-point networks consist of several connections between individual pairs of machines. To go from the source to the destination, a message may have to visit one or more intermediate machines [22]. In this paper point-to-point networks will be considered. Using special structure of the communicating items, such as a regular grid, the points of the network can be addressed by coordinate values. Those points can communicate in a direct way which are neighbors. Points which have a larger distance can communicate through sequences of neighbor points.

In [8,19] communication on the square grid is used. Moreover, in [3,21] communication on the hexagonal grid is investigated, but using only direct (closest) neighbors. A more recent paper considering this grid in communication networks is [1]. The theory of neighborhood sequences is coming from the theoretical image processing, especially from digital

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geometry [5,6,9,23,24]. Higher dimensional (cubic) communicating networks are used in [4]. In this paper the hexagonal (or by other name, the honeycomb) network will be used and some special properties and advantages will be shown.

In [2] agents communicate. We will use similar assumptions: our communicating machines (nodes) are cooperative (they forward the signal to their neighbors; and if a node receives a question and it knows the answer, then it sends the answer in the next step), the communication is error-free, the environment and the network do not change and the communication is synchronous, i.e., there is a global clock-signal.

We will use three coordinate values to refer to the points of the hexagonal grid (using a coordinate frame from [11,21]). Based on the three types of neighbors [7] the theory of neighborhood sequences is developed for the hexagonal network [10,12–15,18]. There is a strange property that the distance based on neighborhood sequences in the hexagonal network may not be symmetric although the neighborhood relations are symmetric. In [13,15] there is a necessary and sufficient condition for neighborhood sequences generating metric distances. In this paper, some other properties of B-distances and digital discs are also discussed and used in applications. A variety of faster relations are defined among neighborhood sequences. Broadcasting a programme and two-way communications with metric, periodic and finite neighborhood sequences are detailed. In the last application the answer signals are really faster than the questions, which property is not present on the square (and other previously used) network-grids. In the next section, as a brief survey, the theory of neighborhood sequences on the hexagonal network is recalled and systematized. Some new definitions and results will also be shown that are used in Section 3 in some applications for communications.

2. Mathematical theory of neighborhood sequences

2.1. Basic definitions and notions

The square grid is usually described by the Cartesian coordinate frame, as $\mathbb{Z}^2$. Then, the usual 4-connected and 8-connected adjacency relations allow to change 1 and 2 coordinates by ± 1, respectively, in a move to a neighbor point/pixel of the grid. We note here that the square grid is self dual, therefore the same description works for both the nodes of the grid and for the pixels of the grid.

The *neighborhood relations* for the triangular grid and for its dual form, the honeycomb grid, are based on the widely used relations (in [7] the relations were showed for the regions of the triangular grid). Similarly, we use three types of neighbors, as Fig. 1 shows, among the nodes of the hexagonal grid. At each point the smallest written number gives the type of neighborhood of the point signed by $\bigcirc$ in the middle. Note that there are two equivalent forms of the triangular grid, in this paper we prefer the 'hexagonal nodes' (i.e., the hexagonal or honeycomb network) form. It is more natural to imagine that the nodes of the grid are people, computers or processors and they can communicate with their neighbors.

In Fig. 1 a node and its 12 neighbors are shown. Only the 1-neighbors are directly connected by a side, i.e., by a gridline, the 2- and 3-neighbors are at the positions of shorter and longer diagonals, respectively. Based on these relations the connectedness is defined in the following way: it is reflexive (i.e., each node is a 1-, 2-, and a 3-connected to itself). Two nodes are 1-connected if they are at most 1-neighbors. Two nodes are 2-connected if they are 2-neighbors or 1-neighbors. The 2-connected node-pairs with the 3-neighbor pairs form the 3-connected relation. So, the m-connectedness relation has increasing and inclusion properties: for $m_1 > m_2$ all m_2-connected nodes of a vertex are its m_1-connected nodes as well.

The coordinate values of the hexagonal network are introduced in [11,21] as it is shown in Fig. 2. Every vertex of the grid is addressed by a unique coordinate triplet. The three values are not independent; their sum is either 0 or 1. The points having 0-sum coordinate values are called *even*, the points with 1-sum are *odd*. One of the first things to notice about the grid is that the points are of two types; the even points have connections in the shape of **Y**, while the odd points have opposite orientations of connections. Since there are two types of points, this grid is not a lattice: some of the grid vectors do not translate the grid to itself. Actually, exactly the vectors described by 0 sum coordinate triplets shift the grid to itself [17].

Using the coordinate triplets one can write the neighborhood and connectedness relations in the following formal form. The points p and q of the hexagonal network are

- m-neighbors ($m = 1, 2, 3$), if the following two conditions hold:
  (1) $|p(i) - q(i)| \le 1$, for $i = 1, 2, 3$,
  (2) $|p(1) - q(1)| + |p(2) - q(2)| + |p(3) - q(3)| = m$.
- m-connected ($m = 1, 2, 3$), if:
  (1) $|p(i) - q(i)| \le 1$, for $i = 1, 2, 3$,
  (2) $|p(1) - q(1)| + |p(2) - q(2)| + |p(3) - q(3)| \le m$.

(These definitions are of the same fashion as similar definitions on the square grid, where $m \in \{1, 2\}$.) We adopt some more definitions from the literature mentioned earlier.

According to the possible types of neighbors, we can define the so-called neighborhood sequences.

The infinite sequence $B = (b(i))_{i=1}^{\infty}$ where $b(i) \in \{1, 2, 3\}$ for all $i \in \mathbb{N}$ is called a *neighborhood sequence*. If for some $l \in \mathbb{N}$, $b(i) = b(i + l)$ holds for every $i \in \mathbb{N}$, then B is called periodic (with period l). In periodic cases we will use the abbreviation $B = (\overline{b(1), \ldots, b(l)})$.