ARTICLE IN PRESS

Discrete Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

# Discrete rigid registration: A local graph-search approach ☆

Phuc Ngo [a,*], Yukiko Kenmochi [b], Akihiro Sugimoto [c], Hugues Talbot [b], Nicolas Passat [d]

[a] *Université de Lorraine, LORIA, France*
[b] *Université Paris-Est, LIGM, CNRS, France*
[c] *National Institute of Informatics, Japan*
[d] *Université de Reims Champagne-Ardenne, CReSTIC, France*

ARTICLE INFO

*Article history:*
Received 2 April 2015
Received in revised form 22 April 2016
Accepted 5 May 2016
Available online xxxx

*Keywords:*
Image registration
Discrete rigid transformation
Discrete optimization
Discrete rigid transformation graph
Graph search

ABSTRACT

Image registration has become a crucial step in a wide range of imaging domains, from computer vision to computer graphics. The core of image registration consists of determining the transformation that induces the best mapping between two images. This problem is ill-posed; it is also difficult to handle, due to the high size of the images and the high dimension of the transformation parameter spaces. Computing an actually optimal solution is practically impossible when transformations are assumed continuous (i.e., defined on $\mathbb{R}^n$). In this article, we initiate the exploration of a new way of considering image registration. Since digital images are basically defined in a discrete framework (i.e., in $\mathbb{Z}^n$), the transformation spaces – despite a potentially high complexity – actually remain finite, allowing for the development of explicit exploration of the parameter space via discrete optimization schemes. We propose an analysis of the very basis of registration, by considering rigid registration between 2D images. We show, in particular, how this problem can be handled in a fully discrete fashion, by computing locally the combinatorial structure of the parameter space of discrete rigid transformations, and by navigating on-the-flight within this space via gradient descent paradigms. This registration framework is applied in real imaging cases, emphasizing the relevance of our approach, and the potential usefulness of its further extension to higher dimension images and richer transformations.



## 1. Introduction

Image processing and analysis applications are involving an increasing amount of images, which result from several total/partial acquisitions of a same scene or different scenes of same semantics, from different viewpoints and/or times. This multiplicity of data related to a same structure of interest has motivated the development, during the last twenty years, of a novel branch of image processing, devoted to image registration [24]. The most frequent application cases of image registration can be found (non-exhaustively) in medical imaging for inter- or intraindividual mapping from multimodal/multitime acquisition [18], in remote sensing for image orthorectification [20], in computer vision for stereovision or augmented reality.

☆ The research leading to these results has received funding from the French *Agence Nationale de la Recherche* (Grant Agreement ANR-2010-BLAN-0205) and the *Programme d'Investissements d'Avenir* (LabEx Bézout, ANR-10-LABX-58).

* Corresponding author. Tel.: +33 354958430; fax: +33 383278319.
*E-mail address:* hoai-diem-phuc.ngo@loria.fr (P. Ngo).

http://dx.doi.org/10.1016/j.dam.2016.05.005
0166-218X/© 2016 Elsevier B.V. All rights reserved.

Basically, registration consists of mapping "at best" a source image $I_s$ onto a target image $I_t$. This implies that $I_s$ and $I_t$ have equal – or at least similar – semantic contents. In general, registration techniques involve searching over the parameter space $\mathbb{T}$ of certain geometric transformations to find the optimal transformation that maps $I_s$ onto $I_t$. In this context, registration is indeed interpreted as an optimization problem. In addition, it is generally modelled as a continuous problem; in other words, $\mathbb{T}$ is considered as a subset of $\mathbb{R}^n$. In particular, this continuous paradigm induces an infinity of possible transformations, and the chosen optimization schemes have to handle this issue.

Our working hypothesis is that, since the considered images are generally discrete, and more precisely digital (i.e., defined in $\mathbb{Z}^k$ mainly with $k = 2$ or 3), a relevant optimization scheme may consist of navigating within the associated (discrete) parameter space of the transformations from $\mathbb{Z}^k$ to $\mathbb{Z}^k$. In particular, even if such parameter space is huge, it may be computed and explored on-the-flight, leading to combinatorial strategies for determining exact locally optimal solutions, or estimating globally optimal ones.

In this article, we initiate the study of this paradigm, by considering the basic case of rigid transformations between 2D images, i.e., images defined on $\mathbb{Z}^2$. In the case of rigid transformations, that are obtained by composition of translations and rotations, the parameter space has three dimensions: one for each principal direction of $\mathbb{Z}^2$, and a third for the rotation angle. The overall parameter space, that is a (finite) subdivision of a subset of $\mathbb{R}^3$, has a polynomial complexity [14], and thus cannot be computed as a whole, in practice. Nevertheless, it can be locally computed as a graph. Such a computation, involving a local exploration of the graph, can be integrated into a gradient descent scheme, which leads to a local optimum, for solving the image registration issue. In particular, a multi-scale scheme is proposed in order to speed-up the optimization process. This study then constitutes a proof of concept for a general combinatorial way of considering image registration.

The remainder of this article – that is an extended and improved version of the conference papers [16,7] – is organized as follows. Section 2 introduces and formalizes the optimization problem of image registration, and our specific purpose, namely discrete image registration. Section 3 describes the parameter space of rigid transformations and the way it can be expressed as a combinatorial structure (named a DRT graph [14]) when handling discrete images. The main contributions of this article can be found in Sections 4 and 5, in which we describe a discrete exploration procedure using neighbouring relations defined on DRT graphs to solve rigid registration issues. Experiments and a conclusion are proposed in Sections 6 and 7, respectively.

## 2. Image registration

In general, registration consists of estimating the optimal spatial transformation so that the source and target images are aligned. In other words, it aims to solve an optimization problem expressed as minimizing a metric error between two images, i.e.

$$\mathcal{T}^* = \arg\min_{\mathcal{T}\in\mathbb{T}} d((I_s \circ \mathcal{T}), I_t) \tag{1}$$

where the objective function $d$ to minimize is the distance (dissimilarity) between images. The transformation $\mathcal{T}^*$ is then searched among the space $\mathbb{T}$ of authorized transformations $\mathcal{T}$, defined via the degrees of freedom on their parameters.

Over the years, many approaches have been proposed to register images, each of which is usually designed for specific applications and types of data. All of them practically rely on the following three key-points.

1. "What": the *distance between images* quantifies how much these images are similar, from a spatial/semantic point of view. In the literature, several distances have been considered for image registration. Depending on the density [6]/sparseness of images, on the one hand, and intensity-based or feature-based [10] paradigms, on the other hand, many distances have been introduced, e.g., cross-correlation, mutual information, ratio image uniformity, least square difference, signed distance, etc. Detailed discussions of these distances can be found, e.g., in [24,3].
2. "Where": the *geometric transformations* that are assumed valid to register the source image onto the target image. The most common transformations are rigid, affine, projective, perspective and global. The determination of these transformations is generally guided by the application and the content of the images. The range is wide, from rigid to non-rigid transformations; the more complex the transformation, the higher the associated parameter space $\mathbb{T}$ (leading to systems that involve from 2 or 3 to thousands of freedom degrees). In addition to geometrical issues, the most complex transformations also induce topological difficulties [17].
3. "How": the *optimization scheme* provides the modus operandi for estimating the optimal distance by exploring the parameter space of geometric transformations. The choice of an optimization technique relies, in particular, on the nature of the distance function and the parameter space of transformations. Various continuous optimization schemes have been employed, such as gradient descent [8], conjugate gradient descent [23], Newton-type methods [2], etc. Due to the ill-condition of the problem, most of these approaches use iterative models of optimization, which consist of performing local search in the parameter space of rigid transformations to find the minima. However, such approaches limit their use in two ways: (1) they strongly rely on the computation of the gradient of the optimizing function, thus this function must be differentiable; (2) they are sensitive to the initial solution and tend to reach local minima. Furthermore, since the problem of image registration is considered in a continuous space, the points of images are assumed to be real values – despite the fact that images are defined in a discrete space. In addition, since the transformation space $\mathbb{T}$ is continuous, all possible rigid transformations cannot be reached. More precisely, a sampling process is necessary to explore the search

Please cite this article in press as: P. Ngo, et al., Discrete rigid registration: A local graph-search approach, Discrete Applied Mathematics (2016), http://dx.doi.org/10.1016/j.dam.2016.05.005