ARTICLE IN PRESS
Discrete Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Note

Counting disjoint hypercubes in Fibonacci cubes

Elif Saygı [a,*], Ömer Eğecioğlu [b]

[a] Faculty of Education, Division of Elementary Mathematics Education, Hacettepe University 06800, Beytepe, Ankara, Turkey
[b] Department of Computer Science, University of California Santa Barbara, Santa Barbara, CA 93106, USA

ARTICLE INFO

Article history:
Received 16 March 2016
Received in revised form 28 June 2016
Accepted 4 July 2016
Available online xxxx

Keywords:
Fibonacci cube
Hypercube
Fibonacci number

ABSTRACT

We provide explicit formulas for the maximum number $q_k(n)$ of disjoint subgraphs isomorphic to the k-dimensional hypercube in the n-dimensional Fibonacci cube Γ_n for small k, and prove that the limit of the ratio of such cubes to the number of vertices in Γ_n is $\frac{1}{2^k}$ for arbitrary k. This settles a conjecture of Gravier, Mollard, Špacapan and Zemljič about the limiting behavior of $q_k(n)$.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

One of the basic models for interconnection networks is the hypercube graph Q_n of dimension n. The vertices of Q_n are represented by binary strings of length n and two vertices are adjacent if and only if they differ in exactly one position. In this model vertex set of the graph denotes the processors and edge set denotes the communication links between processors.

In [4] Fibonacci cubes Γ_n were introduced as a new model of computation for interconnection networks. There is extensive literature on the properties and applications of the Fibonacci cubes. A survey of their usage in theoretical chemistry and summary results on the structure of Fibonacci cubes, including representations, recursive construction, hamiltonicity, the nature of the degree sequence and some enumeration results can be found in the survey [5]. Important properties of Fibonacci cubes in network design are given in [4,2]. The characterization of maximal induced hypercubes in Γ_n was presented in [8]. Many interesting results on the cube polynomial of Γ_n were proved in [6]. A refinement of the cube polynomial of Γ_n in [6] is considered in [10]. In the latter combinatorial interpretation, an extra variable acts as the enumerator of the hypercubes in Γ_n by their distance to the all 0 vertex. Recent papers on additional properties of Fibonacci cubes that have appeared in the literature indicate the continuing interest in these graphs, see for example [1,7,9,11].

Let $q_k(n)$ denote the maximum number of disjoint subgraphs isomorphic to k-dimensional hypercube Q_k in Γ_n. In a recent study several recursive relations and a summation formula for $q_k(n)$ in terms of Fibonacci numbers were presented [3]. Let $|V(\Gamma_n)|$ denote the number of vertices of Γ_n. It was conjectured in [3, Question 3.2] that the limit of the ratio $\frac{q_k(n)}{|V(\Gamma_n)|}$ as n increases without bound exists and is equal to $\frac{1}{2^k}$. In this paper we use the expression for $q_k(n)$ given in [3] in terms of Fibonacci numbers and generating function techniques to derive explicit formulas for $q_k(n)$ for small k. Our computation also gives the form of the $q_k(n)$ for general k, from which it follows that $\lim_{n\to\infty} \frac{q_k(n)}{|V(\Gamma_n)|} = \frac{1}{2^k}$ (Theorem 1).

This paper is organized as follows. In Section 2 we present some useful identities on generating functions of certain subsequences of Fibonacci numbers. We derive our main result in Section 3.

* Corresponding author.
E-mail addresses: esaygi@hacettepe.edu.tr (E. Saygı), omer@ucsb.edu (Ö. Eğecioğlu).

http://dx.doi.org/10.1016/j.dam.2016.07.004
0166-218X/© 2016 Elsevier B.V. All rights reserved.

Please cite this article in press as: E. Saygı, Ö. Eğecioğlu, Counting disjoint hypercubes in Fibonacci cubes, Discrete Applied Mathematics (2016), http://dx.doi.org/10.1016/j.dam.2016.07.004

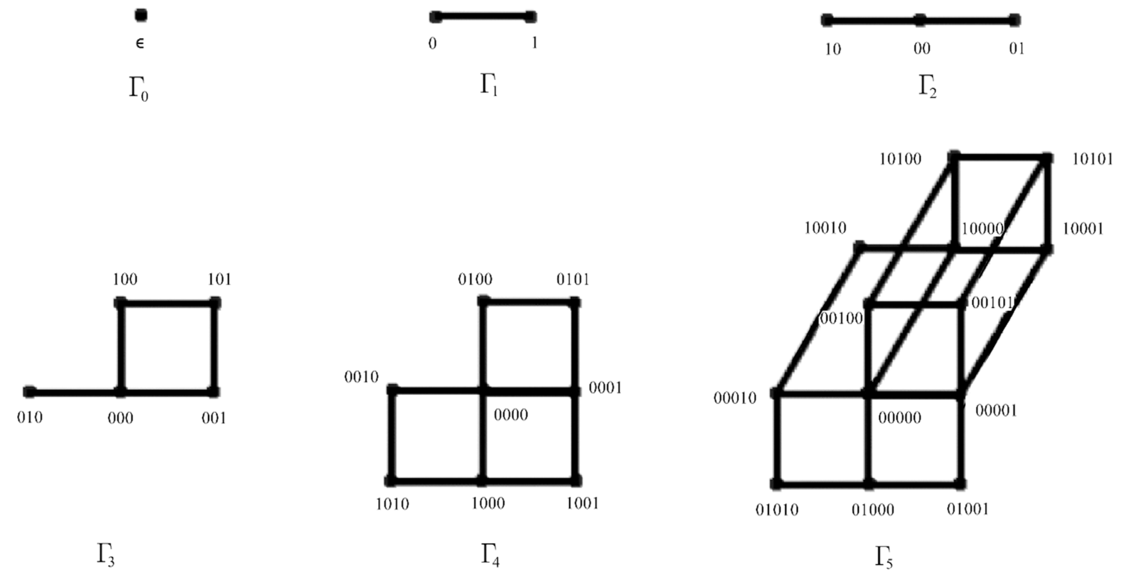

Fig. 1. Fibonacci graphs $\Gamma_0, \Gamma_1, \ldots, \Gamma_5$.

2. Preliminaries

In this section we present some notations and preliminary results related to Fibonacci cubes and Fibonacci numbers.

An n-dimensional hypercube Q_n is the simple graph with vertex set

$$V(Q_n) = \{b_1b_2\cdots b_n \mid b_i \in \{0, 1\},\ 1 \le i \le n\}$$

where the edges are between vertices differing in a single bit. An n-dimensional Fibonacci cube Γ_n is a subgraph of Q_n with vertex set $V(\Gamma_n)$ corresponding to those in Q_n without two consecutive 1s in their string representation. Therefore the vertices of Γ_n have the property that $b_ib_{i+1} = 0$ for all $i \in \{1, 2, \ldots, n-1\}$. For convenience Γ_0 is defined as the graph with a single vertex and no edges.

$V(\Gamma_n)$ is enumerated by Fibonacci number f_{n+2}, where $f_0 = 0, f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \ge 2$. In Fig. 1 we present the first six Fibonacci cubes with their vertices labeled with the corresponding binary strings in the hypercube graph.

Next we consider some special generating functions that we use in the proof of our main result.

If $g(x) = a_0 + a_1x + a_2x^2 + \cdots$ is the generating function of the sequence a_n, then

$$a_0 + a_3x^3 + a_6x^6 + \cdots = \frac{1}{3}\left(g(x) + g(\omega x) + g(\omega^2 x)\right)$$

where ω is a primitive cube root of unity. It follows that for the Fibonacci numbers we have

$$\begin{aligned}\sum_{i\ge 0} f_{3i}x^{3i} &= \frac{1}{3}\left(\frac{x}{1-x-x^2} + \frac{\omega x}{1-\omega x-\omega^2x^2} + \frac{\omega^2 x}{1-\omega^2 x-\omega x^2}\right)\\ &= \frac{2x^3}{1-4x^3-x^6},\end{aligned}$$

and therefore

$$\sum_{i\ge 0} f_{3i}x^{i} = \frac{2x}{1-4x-x^2} \quad \text{and} \quad \sum_{i\ge 0} f_{3i+3}x^{i} = \frac{2}{1-4x-x^2}. \tag{1}$$

Note that (1) is a consequence of the general result that the generating function of the every rth Fibonacci number for $r > 0$ is given by

$$\sum_{i\ge 0} f_{ri}x^{i} = \frac{f_r x}{1-L_r x-x^2}$$

where L_r is the rth Lucas number defined by $L_0 = 2, L_1 = 1$ and $L_r = L_{r-1} + L_{r-2}$ for $r > 1$. However we do not need this expansion in its full generality.

Download English Version:

https://daneshyari.com/en/article/4949932

Download Persian Version:

https://daneshyari.com/article/4949932

Daneshyari.com