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On Monotone Determined Spaces

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Abstract

In this paper, we investigate some basic properties, especially categorical properties, of monotone determined spaces. For a topology τ, we construct a monotone determined topology $md(\tau)$. The main results are: (1) for a space (X, τ), then $md(\tau)$ is the weakest monotone determined topology on X containing τ; (2) the category $\mathbf{Top_{md}}$ of monotone determined spaces with continuous maps is fully co-reflexive in the category $\mathbf{Top}$ of all topology spaces with continuous maps; (3) the category $\mathbf{Top_{md}}$ is cartesian closed.

Keywords: monotone determined space, weak Scott topology, co-reflective, cartesian closed

1 Introduction

For a space X, it is well known that a subset U of X is open iff every net that converges to a point in U is residually in U (cf. [2]). Foe certain order-defined topologies, it suffices to test that criterion for *monotone nets*. Spaces or topologies with that property is called *monotone determined* in [3]. Erné [3] has shown that all locally hypercompact spaces and all Scott spaces are monotone determined, compact open subsets of monotone determined spaces are hypercompact, and a space is

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hypercompactly based iff it is monotone determined and compactly based. It follows that the monotone determined monotone convergence spaces are exactly the Scott spaces of dcpos, the hypercompactly based sober spaces are exactly the Scott spaces of quasialgebraic domains, which gave a negative answer for the question posed by Priestley in [9]: whether there exists a non-quasicontinuous domain on which the Scott topology is spectral.

In this paper, we investigate some basic properties of monotone determined spaces, especially the categorical properties. For a topology τ, we construct a monotone determined topology $md(\tau)$. It is shown that $md(\tau)$ is the weakest monotone determined topology on X containing τ and $cl_\tau D = cl_{md(\tau)} D$ for any directed subset D. Let $\mathbf{Top_{md}}$ be the category of monotone determined spaces with continuous maps and $\mathbf{Top}$ the category of all topology spaces with continuous maps. We show that the product and the limit of monotone determined spaces are $md(\prod_{i\in I} X_i)$ and $md(\varprojlim_j X_j)$, respectively, where $\prod_{i\in I} X_i$ and $\varprojlim_j X_j$ are the product and the limit in $\mathbf{Top}$, respectively. It is proved that $\mathbf{Top_{md}}$ is fully co-reflexive in $\mathbf{Top}$ and $\mathbf{Top_{md}}$ is cartesian closed.

2 Preliminaries

In this section we recall some basic definitions and notations used in this note, more details can be found in [1,5,8]. Let P be a poset, $x \in P, A \subseteq P$. Let $\uparrow x = \{y \in P : x \leq y\}$ and $\uparrow A = \{y \in P : x \leq y$ for some $x \in A\}$, $\downarrow x$ and $\downarrow A$ are defined dually. A is said to be an *upper set* if $A = \uparrow A$. $A^\uparrow$ and $A^\downarrow$ denote the sets of all upper and lower bounds of A, respectively. Let $A^\delta = (A^\uparrow)^\downarrow$. P is said to be a *directed complete poset*, a dcpo for short, if every directed subset of P has the least upper bound in P. The *Alexandroff topology* $A(P)$ on P is the topology consisting of all its upper subsets. The topology generated by the collection of sets $P\backslash\downarrow x$(as subbasic open subsets) is called *upper topology* and denote by $\nu(P)$. A subset U of P is called *Scott open* if $U = \uparrow U$ and $D \cap U \neq \emptyset$ for all directed sets $D \subseteq P$ with $\vee D \in U$ whenever $\vee D$ exists. The topology formed by all the Scott open sets of P is called the *Scott topology*, written as $\sigma(P)$.

We order the collection of nonempty subsets of a poset P by $G \leq H$ if $\uparrow H \subseteq \uparrow G$. We say that a nonempty family of sets is directed if given F_1, F_2 in the family, there exists F in the family such that $F_1, F_2 \leq F$, i.e., $F \subseteq \uparrow F_1 \cap \uparrow F_2$. For nonempty subsets F and G of a dcpo L, we say F approximates G if whenever a directed subset D satisfies $\vee D \in \uparrow G$, then $d \in \uparrow F$ for some $d \in D$. A dcpo L is called a *quasicontinuous domain* if for all $x \in L$, $\uparrow x$ is the directed (with respect to reverse inclusion) intersection of sets of the forms $\uparrow F$, where F approximates $\{x\}$ and F is finite.

Give a topological space (X, τ), define an order $\leq_\tau$, called the *specialization order*, by $x \leq_\tau y$ if and only if $x \in cl_\tau\{y\}$. Clearly, each open set is an upper set and each closed set is a lower set with respect to the specialization order $\leq_\tau$. Denote the closure of subset $A \subseteq X$ by $cl_\tau A$ and interior of A by $int_\tau A$ in (X, τ).

Definition 2.1 ([3]) A space is called *locally hypercompact* if for any $x \in X$ and