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Hybrid and Subexponential Linear Logics

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Abstract

HyLL (Hybrid Linear Logic) and SELL (Subexponential Linear Logic) are logical frameworks that have been extensively used for specifying systems that exhibit modalities such as temporal or spatial ones. Both frameworks have linear logic (LL) as a common ground and they admit (cut-free) complete focused proof systems. The difference between the two logics relies on the way modalities are handled. In HyLL, truth judgments are labelled by *worlds* and hybrid connectives relate worlds with formulas. In SELL, the linear logic exponentials (!, ?) are decorated with labels representing *locations*, and an ordering on such labels defines the provability relation among resources in those locations. It is well known that SELL, as a logical framework, is strictly more expressive than LL. However, so far, it was not clear whether HyLL is more expressive than LL and/or SELL. In this paper, we show an encoding of the HyLL's logical rules into LL with the highest level of adequacy, hence showing that HyLL is as expressive as LL. We also propose an encoding of HyLL into SELL$^{\Cap}$ (SELL plus quantification over locations) that gives better insights about the meaning of worlds in HyLL. We conclude our expressiveness study by showing that previous attempts of encoding Computational Tree Logic (CTL) operators into HyLL cannot be extended to consider the whole set of temporal connectives. We show that a system of LL with fixed points is indeed needed to faithfully encode the behavior of such temporal operators.

Keywords: Linear Logic, Hybrid Linear Logic, subexponentials, logical frameworks, Temporal Logic.

1 Introduction

Logical frameworks are adequate tools for specifying proof systems, since they support levels of abstraction that facilitate writing declarative specifications of object-logic proof systems. Many frameworks have been used for the specification of proof systems, and linear logic [13] (LL) is one of the most successful ones. This is mainly because LL is resource conscious and, at the same time, it can internalize classical and intuitionistic behaviors (see, for example, [6, 14]).

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However, since specifications of object-level systems into the logical framework should be natural and direct, there are some features that cannot be adequately captured in LL, in particular modalities different from the ones present in LL.

Extensions of LL have been proposed in order to fill this gap. The aim is to propose stronger logical frameworks that preserve the elegant properties of linear logic as the underlying logic. Two of such extensions are HyLL (Hybrid Linear Logic)[3] [8] and SELL (Subexponential Linear Logic) [9,19]. These logics have been extensively used for specifying systems that exhibit modalities such as temporal or spatial ones. The difference between HyLL and SELL relies on the way modalities are handled.

In HyLL, truth judgments are labeled by worlds and two hybrid connectives relate worlds with formulas: the satisfaction `at` which states that a proposition is true at a given world, and the localization $\downarrow$ which binds a name for the (current) world the proposition is true at. These constructors allow for the specification of modal connectives such as $\Box A$ (A is true in all the accessible worlds) and $\Diamond A$ (there exists an accessible world where A holds). The underlying structure on worlds allows for the modeling of transitions systems and the specification of temporal formulas [8, 10].

In SELL, the LL exponentials (!, ?) are decorated with labels: the formula $?^a A$ can be interpreted as A *holds in a location*, *modality*, or *world* a. Moreover, A can be deduced in a location b related to a ($b \preceq a$). On the other side, the formula $?^a!^a A$ means that A *is confined into the location* a, that is, the information A is not propagated to other worlds/locations related to a. While linear logic has only seven logically distinct prefixes of bangs and question-marks (none, !, ?, !?, ?!, !?!, ?!?), SELL allows for an unbounded number of such prefixes (e.g., $!^a?^c?^d$). Hence SELL enhances the expressive power of LL as a logical framework.

Up to now, it was not clear how HyLL is related to LL and/or SELL. In this paper we answer that question by showing a direct encoding of the HyLL's logical rules into LL with the highest level of adequacy. Hence, we show that HyLL is actually as expressive as LL.

We also propose an encoding of HyLL into SELL$^{\Cap}$ (SELL with quantification over locations) that gives better insights about the meaning of worlds in HyLL. More precisely, we represent HyLL formulas as formulas in SELL and encode the logical rules as formulas in SELL$^{\Cap}$. We show that a flat subexponential structure is sufficient for representing any world structure in HyLL. This explains better why the worlds in HyLL do not add any expressive power to LL: they cannot control the logical context as the subexponentials do with the promotion rule.

HyLL has been shown to be a flexible framework for the specification of biological systems [10] where both the system and its properties are specified using the same logic. More precisely, the properties of interest are first written in Computational Tree Logic (CTL) and later encoded as HyLL formulas. However, there was no a formal statement about the CTL fragments that can be adequately captured

[3] Actually, HyLL is an extension of intuitionistic linear logic (ILL), while SELL can be viewed as an extension of both ILL or LL.