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Stochastic Pruning and Its Application for Fast Estimation of the Expected Total Output of Complex Systems

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Abstract

A powerful method referred to as *stochastic pruning* is introduced for analysing the performance of common complex systems whose component failures follow a homogeneous Poisson process. The method has been applied to create a very fast solver for estimating the production availability of large repairable flow networks with complex topology. It is shown that the key performance measures *production availability* and *system reliability* are all properties of a stochastically pruned network with corresponding pruning probabilities. The high-speed solver is based on an important result regarding the average total output of a repairable system including components characterised by constant failure/hazard rates. The average output over a specified operation time interval is given by the ratio of the expected momentary output of the stochastically pruned system, where the separate components are pruned with probabilities equal to their unavailabilities, and the maximum momentary output in the absence of component failures. The running time of the algorithm for determining the expected total output of the system over a specified time interval is independent of the length of the operational interval and the failure frequencies of the edges. The high-speed solver has been embedded in a software tool, with graphics user interface by which a flow network topology is drawn on screen and the parameters characterising the edges and the nodes are easily specified. The software tool has been used to analyse a gas production network and to study the impact of the network topology on the network performance. It is shown that two networks built with identical type and number of components may have very different performance levels, because of slight differences in their topology.

Keywords:
stochastic pruning, stochastic flow networks, production availability, repairable flow networks, performance, software tool, simulation

1 Introduction

Stochastic systems where the component failures are random events are very common and have been discussed extensively in the system reliability literature related to reliability networks ([1,11,5]). Stochastic systems where the flow capacities of components are random variables have also been considered in the literature dealing with stochastic flow networks ([7,2,10,9,16]) The problem of interest was the probability that, on demand, the throughput flow will be equal or greater than a specified level. Evaluating the reliability of complex systems and maximising the

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flow in stochastic flow networks has been traditionally based on methods involving minimal cut sets or minimal paths. A similar approach has been adopted by Jane et al. (1993), where stochastic flow networks with multistate components have been considered. Minimum cut sets have also been used by Fishman (1987) to evaluate the distribution of the maximum flow in a directed network whose edges have random capacities. Although, for small-size reliability networks and flow networks, an approach based on minimal paths or minimal cut sets is acceptable, with increasing the size of the network, the number of minimal paths and cut sets increases exponentially and this approach is no longer feasible. This point has been illustrated with the example in Figure 1 discussed in [14]. The flow network in the figure has $N^N + N$ minimal cut sets and N^{N+1} minimal paths. Even for the moderate $N = 10$, the storage and manipulation of the minimal paths and cut sets is impossible. As a result, an algorithm based on determining all minimal paths or cut sets is very inefficient because it will run *in exponential time*.

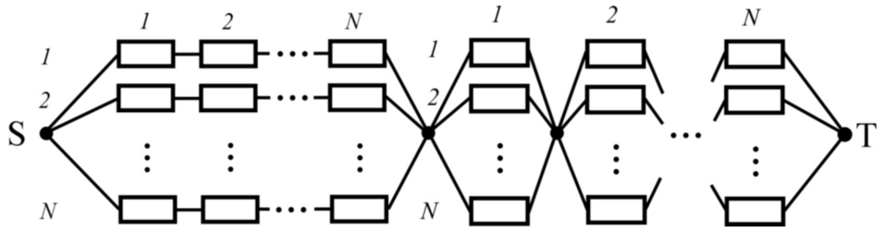

Fig. 1. An example of a flow network where the number of minimal paths and minimal cut sets increases exponentially with increasing the size of the network [14].

A key performance measure of many complex systems over a specified time interval is the *production availability*. This is the expected fraction of total system output during a specified time interval, in the presence of component failures. It is defined as the ratio

$$\psi = \frac{\overline{Q}_T}{Q_{T0}} \qquad (1)$$

of the total expected output $\overline{Q}_T$ of the system in the presence of component failures during a specified time interval [0,a] and the total output Q_{T0} of the system that could be obtained in the absence of any component failures. For repairable flow networks (e.g. gas production networks), $\overline{Q}_T$ in equation (1) is the expected maximum throughput flow in the presence of failures of components building the network and Q_{T0} is the total throughput flow that could be obtained during this time interval, in the absence of component failures (Fig.2). For a system of electrical generators connected to an electrical grid, $\overline{Q}_T$ in equation (1) is the expected electrical energy produced by the generators over a specified time interval in the presence of failed generators and Q_{T0} is the total amount of electrical energy that could be produced during this time interval, in the absence of any failed generators. To reveal the variation of the total output, a large number of failure-repair histories during the period of operation of the system must be simulated (Fig.2a).

Upon a component failure, the output of the system usually decreases (Fig.2b). Such are for example the very common systems consisting of interconnected sources