Robustness of f- and g-generated Fuzzy (Co)Implications: The Yager's (Co)Implication Case Study

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Abstract

This paper studies the robustness of intuitionistic fuzzy implications in fuzzy reasoning based on Atanassov's intuitionistic fuzzy logic. Starting with an evaluation of the sensitivity in representable fuzzy negations, we apply the results in the Yager's classes of fuzzy implications called the f- and g-generated fuzzy implications. The paper formally states that the robustness preserves the projection functions in such class and also discusses their corresponding dual operators.

Keywords: Robustness analysis, Intuitionistic fuzzy logic, Yager's implications, f- and g-generated implications.

1 Introduction

Since Yager's classes of fuzzy implications called the f- and g-generated implications [24] have been used in common sense reasoning, there is a practical need for intuitionistic fuzzy versions of these operations, i.e., an operation $I_f(x, y)$ that uses the membership degrees $\mu_A(u) = a$ and $\mu_B(u) = b$ of two intuitionistic fuzzy sets A and B to estimate the uncertainty degree of confidence in the statement

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$A \to_f B$. These operations are also extensions of the corresponding crisp opera-
tion: $I_f(0,1)=I_f(0,0)=1$, $I_f(1,0)=0$ and $I_f(1,0)=1$.

The concepts of maximum and average perturbations of fuzzy sets [25], estimat-
ing the maximum and average perturbation parameters for various methods of fuzzy
reasoning is relevant for systems based on fuzzy logic (FL) and as a consequence
for intuitionistic fuzzy logic (IFL).

1.1 Main related works

In [15], Li et al. study properties of some measures of robustness (or sensitivity)
of fuzzy connectives and implication operators and discuss their relationships with
perturbation properties of fuzzy sets. Many other works have discussed the robust-
ness analysis also including the δ-sensitive approach, see e.g. [14], [15], [16], [17] and
[18].

This paper extends the δ-sensitivity study of some intuitionistic fuzzy con-
nectives (IFCs) according with results previously presented in [15], based on
Atanassov's Intuitionistic Fuzzy Logic (A-IFL), as presented in [1].

In [24], some properties of the Yager's classes of fuzzy implications including
the h-generated implications are discussed, describing their relationships amongst
themselves and with the well established strong and residual implication classes [9].

Additionally, in [19], the Bandler-Kohout subproduct relational inference sys-
tem with the fuzzy implication interpreted as the Yager's classes of implications
are reported, studying many of the desirable properties as interpolativity, conti-
nuity, robustness and computational efficiency, expanding the choice of operations
available to practitioners.

A semantic behaviour of a fuzzy rule model is proposed in [13] as a pair of
fuzzy implication and modus ponens generating function used for inference. Such
methodology is applied to Yager's models which are obtained from Yager implication
function. By Yager's implicative implication, it is shown to be midway between the
usual residual and strong implications generated from the product t-norm. In fact,
Yager's implication belongs to a more general family of implications that can also
be generated from the t-norms.

Such analysis can improve the study of the stability of systems based on intu-
itionistic fuzzy rules. The notion of δ-sensitivity of fuzzy connectives in the fuzzy
intuitionistic approach, which is characterized by the non-complementary relation-
ship between the membership and non-membership functions, as proposed in [2], is
considered in this work.

1.2 Main contribution of the paper

Following preliminary studies introduced in [21], this paper considers the robustness
analysis defined on δ-sensitivity of the Atanassov intuitionistic fuzzy approach of
the Yager's implication classes, the f- and g-generated implications [24], focusing
on their pointwise components obtained by the projections related to membership
and non-membership functions.