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Clear and smooth path planning

Mansoor Davoodi [a,*], Fatemeh Panahi [b], Ali Mohades [c], Seyed Naser Hashemi [c]

[a] Department of Computer Sciences and Information Technology, Institute for Advances Studies in Basic Sciences, Zanjan, Iran
[b] Group of Games and Virtual Worlds, Department of Information and Computing Sciences, Utrecht University, The Netherlands
[c] Laboratory of Algorithms and Computational Geometry, Department of Mathematics and Computer Science, Amirkabir University of Technology, Tehran, Iran

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ABSTRACT

One of the challenging problems in motion planning is finding an efficient path for a robot in different aspects such as length, clearance and smoothness. We formulate this problem as two multi-objective path planning models with the focus on robot's *energy consumption* and path's *safety*. These models address two five- and three-objectives optimization problems. We propose an evolutionary algorithm for solving the problems. For efficient searching and achieving Pareto-optimal regions, in addition to the standard genetic operators, a family of path refiner operators is introduced. The new operators play a local search role and intensify power of the algorithm in both explorative and exploitative terms. Finally, we verify the models and compare efficiency of the algorithm and the refiner operators by other multi-objective algorithms such as strength Pareto evolutionary algorithm 2 and multi-objective particle swarm optimization on several complicated path planning test problems.

1. Introduction

Path planning (PP) problem is one of the challenging problems in motion planning which has widespread applications in robotics and manipulation and in daily routine works. The goal of this problem is to look for an optimal path between two given points—called *source* and *destination* denoted by *s* and *d*, respectively—in a workspace contains some obstacles. Since there are many types of robots with different abilities and constraints, and different applications, it is nearly impossible to provide a unique exact definition for the expression “optimal path” in the robot path planning context. But generally, it can be focused on two aspects of a plan: *energy* and *safety*. The energy is an issue related to the physical and mechanical structure of the robot and its abilities for moving, rotating and sensing. The safety is related to the planned path as it is very important to safe both robot and obstacles in the presence of some unavoidable errors raised due to sensors' imperfections, finite computations, actuators' inaccuracies or uncertainty in the workspace [1].

With regard to the complexity of PP problems which is NP-hard [2] and also widespread applications of them, such as computer games and animations, bio-informatics, assembly planning and manufacturing design [3–5], many exact, approximation and heuristic algorithms have been proposed to solve the problem. Most of these algorithms focused on minimizing the path's length [1,6–8]. Cell decomposition, roadmap, potential field and heuristic algorithms can be reviewed as some of such algorithms. Cell decomposition methods decompose the workspace and convert the PP problem to a graph searching problem [8–10]. If the workspace is a path-connected space, these methods guarantee to find a feasible path (a path which starts from *s* and ends at *d* without crossing the obstacles), but there is no guarantee on the other aspects of the optimality. Roadmap approaches first construct a set of collision-free connected *highways* in the workspace and then for a given pair *s* and *d*, connect them to the highways. Finally the shortest *s*–*d*–path can be found in such highways. *Visibility graph* (VG) and *Voronoi diagram* (VD) are two famous roadmap objects [11,12]. VG is a graph whose vertices are obstacle vertices (and points *s* and *d*). There exists an edge between two nodes in VG if and only if they are *visible* to each other in the workspace. If the weight of each edge is equal to its length, VG is a weighted connected graph and we can find the minimum length path by searching the shortest path on the graph. Unfortunately, the optimal path in VG is not completely collision free; it is *semi-collision* free. In other words, it is possible that some parts of path lie on the boundary of obstacles. So, in these approaches usually the minimum distance between path and obstacle—called *confidence interval* or *minimum clearance*—is zero. On the other hand, by using VD approach a path with the maximum confidence interval can be found, but in most cases this path is too long for being reasonable. VD of a set of

* Corresponding author. Tel.: +98 24 3315 3376;
fax: +98 24 3315 3341/14155071.
E-mail addresses: mdmonfared@iasbs.ac.ir (M. Davoodi), F.Panahi@uu.nl (F. Panahi), mohades@aut.ac.ir (A. Mohades), nhashemi@aut.ac.ir (S.N. Hashemi).

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obstacles is defined as locus of all points in the workspace which has at least two nearest obstacles. Also, a hybrid of generalized Voronoi diagram and randomized planning was presented for translating and rotating rigid bodies [13].

Potential field approach is a fast heuristic search method for PP problem but it is possible that this method is trapped in local optimums [8]. This method uses two differentiable *attractive* and *repulsive* functions simultaneously. The first function attracts robot to the destination point and the second one repels it from obstacles. The efficiency of potential field algorithm is dependent seriously on the functions quality. Several mathematical definitions have been presented to implement the functions [8]. There are also some PP algorithms which depend less on the dimension of search space; sample-based methods like probabilistic roadmap and other variations of it [1,14] and combinatorial heuristic approaches such as particle swarm optimization and ant colony optimization [15], can be mentioned. Also, genetic algorithms (GA) as robust popular heuristic search methods have been extensively used to solve the PP problem in the discrete and continuous spaces [16–18].

Each of the above approaches has some advantages and disadvantages, but none of them regards all the mentioned aspects of the path's optimality, energy and safety, simultaneously. In fact, most of the methods consider just one of the objectives length, clearance or smoothness. However, to achieve an ideal path, it is necessary to synthesize all of these objectives. This challenge is introduced as *natural-looking path* in the Wein's study [19]; a path which is short and smooth and keeps a predefined amount of distance from obstacles. Two objects VG and VD are combined and a new object, *visibility-Voronoi diagram* with clearance c (denoted by $VV^{(c)}$-diagram) is constructed. The extension of $VV^{(c)}$-diagram to higher dimensional space has not been studied, but with regard to the complexity of VG and VD in higher dimensions, it can be said that it is exponentially complicated. Also, some studies have tried to achieve the safety term by using *Minkowski sum*, or by doing some post-processing on the output of other algorithms in order to improve it in terms of other aspects of optimality [20,21] e.g. first a sample-based algorithm finds a piece-wise short path, and then a post-process tries to improve smoothness or clearance of the path. For example, Pan et al. [22] by focusing on translation and rotation motions used cubic B-spline trajectories and presented an efficient post processing algorithm to generate smooth paths by sample-based methods. Unfortunately, in these cases there is no guarantee the path remains short after performing post-process. Also, as mentioned in [19], these types of algorithms are inefficient in higher dimensions as well as in complicated search spaces such as narrow passage and clutter workspaces.

Castillo et al. [23] studied the PP problem in a grid with two objective functions, length and difficulty. They considered a predefined difficulty weight to each cell of the grid. Their algorithm uses a simple ranking procedure based on non-dominated solutions to sort the population with respect to the length and difficulty. For a grid workspace, Ahmed and Deb [24] considered the problem of offline point-to-point path planning with travel distance, safety and smoothness. They used non-dominated sorting genetic algorithm II (NSGA-II) with several chromosome representations for monotone paths. Also, they used spline representation to generate piece-wise polynomial and B-spline smooth paths on the grid [25]. Piazzi et al. [26] used the polynomial curves and η^3-splines and proposed an approach to find smooth and short paths. Elshamli et al. [27] provided a single objective model based on the linear combination of all goal functions. This technique has some significant defects, e.g. there is no proportion between the objective space and weighted space, specially in non-uniform search spaces. In addition, there is no exact information about the priority of the objectives [28].

Since population based evolutionary algorithms can emphasize to all objectives simultaneously, they are appropriate candidates to find a set of Pareto-optimal solutions as diverse as possible in objective space. NSGA-II and strength Pareto evolutionary algorithm 2 (SPEA2) are two robust multi-objective evolutionary approaches [29,30] that have been tested by several real and benchmark problems [28]. Mittal and Deb [31] used NSGA-II for offline path planning of unmanned aerial vehicles by using B-spline curves. They considered the objectives of minimizing length and maximizing margin of safety. Zhang et al. [32] formulated the problem of path planning under uncertainty as a bi-objective optimization problem focusing on minimizing risk degree and path's length, and used multi-objective particle swarm optimization (PSO) to solve it. Similarly, a hybrid of PSO and gravitational search algorithm was proposed by Purcaru et al. [33] for finding short and safe paths in dangerous environments. Davoodi et al. [34] regarded the path planning in a grid and considered two objectives, minimizing the path length in terms of Manhattan distance with four-connectivity and eight-connectivity and maximizing the distance of path from obstacle.

Most of practical applications of PP need to a continuous workspace, e.g. car like robots. In this paper we suggest two multi-objective PP models on the continuous space focusing on minimizing energy consumption and maximizing path's safety. To achieve these goals, we follow three objectives: minimizing path length, maximizing smoothness, and maximizing clearance. Each of the models can be used in different applications with regard to the availability of minimum necessary confidence interval. Based on authors' knowledge there is no study in the literature that simultaneously focuses on these objectives in the continuous space for point-to-point path planning. However, similar existing studies, for example [24–27], considered just some simple and usual workspaces for testing their presented algorithms. In this paper, for efficiently solving the multi-objective path planning problems, we propose an evolutionary algorithm based on the NSGA-II framework such that in addition to the standard genetic operators, it benefits a family of robust geometric path refiner operators. These new operators play a local search role and increase exploration and exploitation power of the algorithm. Also, this family of operators can be used in the body of any heuristic search algorithm for PP. We test our algorithm and the new operators, and show that these operators significantly improve the efficiency of such algorithms in the complicated PP workspaces such that the algorithm is able to find near Pareto-optimal solutions in the most of simulations.

This paper includes six sections. Section two presents two mathematical multi-objective formulations of PP problem. Section three briefly reviews multi-objective optimization concepts. Genetic operators, new refiner operators and the algorithm are proposed in the fourth section. Section five shows simulation and comparison results, and finally we draw a conclusion in the last section.

2. Mathematical modeling

For a given path planning workspace contains some polygonal obstacles and two source and destination points s and d, we aim to find an energy and safety terms optimal point-to-point path between s and d which does not cross the obstacles. In most of applications, robot spends energy for sensing and moving which is performed by wheels or legs. Furthermore, usually robot's rotations along the path needs to energy because of its mechanical and physical construction. Therefore, the path's length and smoothness can be used as two appropriate factors for consuming energy. In the mathematic literatures a smooth path is a differentiable path, but since our discussion is based on the point-to-point planning strategy, we focus on the number of rotations and amount of them for smoothness objective. More precisely, to achieve the maximum