Information and Computation ••• (••••) •••–•••

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Information and Computation

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Classifying recognizable infinitary trace languages using word automata

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A R T I C L E   I N F O

Article history:
Received 16 December 2014
Available online xxxx

Keywords:
Mazurkiewicz traces
Omega-regular languages
Classification
Finite state automata

A B S T R A C T

We address the problem of providing a Borel-like classification of languages of infinite Mazurkiewicz traces, and provide a solution in the framework of ω-automata over infinite words – which is invoked via the sets of linearizations of infinitary trace languages. We identify trace languages whose linearizations are recognized by deterministic weak or deterministic Büchi (word) automata. We present a characterization of the class of linearizations of all recognizable ω-trace languages in terms of Muller (word) automata. Finally, we show that the linearization of any recognizable ω-trace language can be expressed as a Boolean combination of languages recognized by our class of deterministic Büchi automata.

1. Introduction

Traces were introduced as models of concurrent behaviors of distributed systems by Mazurkiewicz, who later also provided an explicit definition of infinite traces [9]. Zielonka demonstrated the close relation between traces and trace-closed sets of words, which can be viewed as "linearizations" of traces, and established automata-theoretic results regarding recognizability of languages of finite traces [13] (cf. also [5,8]). Later, [6,4,10] enriched the theory of recognizable languages of infinite traces (*recognizable ω-trace languages*), by introducing models of computations viz. asynchronous Büchi automata and deterministic asynchronous Muller automata. Being closely related to word languages, a set of infinite traces is recognizable iff the corresponding trace-closed set of infinite words is.

In the case of ω-regular word languages, there exists a straightforward characterization of languages recognized by deterministic Büchi automata, and a result due to Landweber states that it is decidable whether a given ω-regular language is deterministically Büchi recognizable [11, Chapter 1]. However, analogous results over recognizable ω-trace languages have only recently been established in terms of "synchronization-aware" asynchronous automata [2].

While asynchronous automata are useful in implementing distributed monitors and distributed controllers, their constructions are prohibitively expensive even by automata-theoretic standards. On the other hand, for applications like model-checking and formal verification, word automata recognizing trace-closed languages would already allow for analysis of most of the interesting properties pertaining to distributed computations.

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[1] Supported by the DFG Research Training Group-1298 AlgoSyn.
[2] Supported by the CASSTING Project funded by the European Commission Seventh Framework Programme, Grant number 601148.

http://dx.doi.org/10.1016/j.ic.2017.04.006
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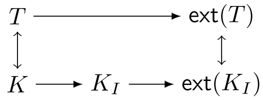
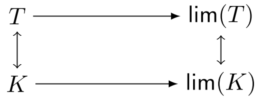


(a) The correspondence of infinitary extensions for all regular trace languages T.



(b) The correspondence of infinitary limits for all *limit-stable* trace languages T.

Fig. 1. From trace-closed regular languages to trace-closed ω-regular languages.

Therefore, in this paper, we study classes of ω-regular word languages that allow us to "transfer" interesting results to the corresponding classes of recognizable ω-trace languages. In particular, motivated by the Borel hierarchy for regular languages of infinite words, our main contribution is a new setup for a classification theory for recognizable ω-trace languages in terms of trace-closed, ω-regular word languages. The results of this paper have been announced in [3].

Recall that in the sequential setting, reachability languages and deterministically Büchi recognizable languages – constituting the lowest levels of the Borel hierarchy – can be obtained via natural operations over regular languages $K \subseteq \Sigma^*$ in the following ways:

- $\mathsf{ext}(K) = K \cdot \Sigma^\omega = \{\alpha \in \Sigma^\omega \mid \alpha$ has a prefix in $K\}$
- $\mathsf{lim}(K) = \{\alpha \in \Sigma^\omega \mid \alpha$ has infinitely many prefixes in $K\}$

These operations, which we call the *infinitary extension* and the *infinitary limit* of K, can be generalized to obtain infinitary extensions $\mathsf{ext}(T)$ and infinitary limits $\mathsf{lim}(T)$ of regular trace languages T.

In this paper, given the trace-closed word language K corresponding to a regular trace language T, we firstly show that K can be modified to K_I such that $\mathsf{ext}(K_I)$ is also trace-closed and corresponds to the linearization of $\mathsf{ext}(T)$ (here I denotes the independence relation over the alphabet Σ). Building on this, we are able to characterize the class of Boolean combinations of languages $\mathsf{ext}(T)$ as precisely those whose linearizations are recognized by the class of "I-diamond" deterministic weak automata (DWAs).

Next, we consider infinitary limits. Here the situation is different, in that there exist regular trace languages T such that although the trace-closed word language L corresponding to $\mathsf{lim}(T)$ is ω-regular, it is not recognized by any I-diamond deterministic Büchi automaton (DBA). We therefore introduce the class of *limit-stable* word languages K – and by extension limit-stable trace languages T – such that the correspondence of Fig. 1b holds, and $\mathsf{lim}(K)$ can be characterized in terms of I-diamond DBA.

It is well known that every trace-closed ω-regular language is recognized by an I-diamond Muller automaton [4]. We characterize these languages in terms of a well defined class of I-diamond Muller automata. And lastly, justifying our definitions, we show that every trace-closed ω-regular word language (that is, the linearization of any recognizable ω-trace language) can be expressed as a finite Boolean combination of languages $\mathsf{lim}(K)$, with K limit-stable.

In related work, Diekert & Muscholl [4] consider a form of "deterministic" trace languages. It is shown that every recognizable language of infinite traces is a Boolean combination of these deterministic languages. However, in the attempt to characterize the corresponding "deterministic" trace-closed word languages in terms of I-diamond automata, it is necessary to extend the Büchi acceptance condition beyond what we know from standard definitions [10]. It had been left open in [10] whether there exists a class of deterministic asynchronous Büchi automata for deterministic trace languages.

We begin with presenting the basic definitions and notations. In Sec. 3, we present the operations ext and lim that allow for construction of recognizable ω-trace languages from regular trace languages. In particular, we exhibit recognizable ω-trace languages whose linearizations are recognized by I-diamond DWA, and those whose linearizations are I-diamond DBA recognizable. Finally, we establish our main result demonstrating the expressiveness of I-diamond DBA recognizable trace-closed languages.

2. Preliminaries

We denote a recognizable language of finite words, or simply a *regular language*, with the upper case letter K and a class of such languages with $\mathcal{K}$. Finite words are denoted with lower case letters u, v, w etc. Infinite words are denoted by lower case Greek letters α and β, and a recognizable language of infinite words, or simply an *ω-regular language*, by upper case L. For a word u or α, we denote its infix starting at position i and ending at position j by $u[i, j]$ or $\alpha[i, j]$, and the i^{th} letter with $u[i]$ or $\alpha[i]$. For a language K, we define $\overline{K} := \Sigma^* \setminus K$.

Please cite this article in press as: N. Chaturvedi, M. Gelderie, Classifying recognizable infinitary trace languages using word automata, Inf. Comput. (2017), http://dx.doi.org/10.1016/j.ic.2017.04.006