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Quantum game players can have advantage without discord

Zhaohui Wei [a,*], Shengyu Zhang [b]

[a] School of Physics and Mathematical Sciences, Nanyang Technological University and Centre for Quantum Technologies, Singapore
[b] Department of Computer Science and Engineering, The Chinese University of Hong Kong, Hong Kong

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ABSTRACT

In the study of information processing tasks, the ultimate question is when quantum advantage exists and, if existing, how much it could be. In a broad class of quantum scenarios, the correlation among different parties involved is a key factor to quantum efficiency under concern. This correlation is usually quantified by entanglement or discord, which are widely considered as important or even necessary resources for quantum advantage to exist. In this paper, we examine a problem of this nature in the realm of game theory. We exhibit a natural zero-sum game, where in a certain classical equilibrium, a situation in which no player can increase her payoff by any local classical operation, whoever switches to a quantum computer has a big advantage over its classical opponent. The equilibrium state is symmetric, thus fair to both players, but the state as a shared correlation has zero entanglement or discord.

Requested note on comparison with the conference version: This submission added a formal definition of and some references on discord. Section 3.3 has been completely rewritten, adding a formulation of the problem of finding the maximum quantum gain for Dice Matching game, followed by a different and simplified proof of the same optimality result as previous.

1. Introduction

Quantum computers have exhibited tremendous power in algorithmic, cryptographic, information theoretic, and many other information processing tasks, compared with their classical counterparts. Meanwhile, for a large number of problems, quantum computers are not able to offer much advantage over classical ones. When and why quantum computers are more powerful are always at a central position in studies on quantum computation and quantum information processing. A particularly interesting class of scenarios is when there are, implicitly or explicitly, at least two parties involved who share a state, the correlation in this state is the key factor. What accounts for the quantum advantage is often *entanglement*, one of the most distinctive characters of quantum information. Indeed, it has been shown that a quantum algorithm with only

* Corresponding author.
E-mail address: weizhaohui@gmail.com (Z. Wei).

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slight entanglement can be simulated efficiently by a classical computer [19]. In certain potential applications of quantum algorithms, it is also shown that entangled measurement is necessary for the existence of efficient quantum algorithms [9].

Recently it was realized that entanglement is not always a necessary resource needed for generating quantum correlations. It has been found that *discord*, another unique character of quantum states, also plays an important role in quantum information processing [16]. Discord is a relaxed version of entanglement—states with positive entanglement must also have positive discord, but there are states with positive discord but zero entanglement. Cases have been discovered where non-classical correlations can be generated from states with zero entanglement but nonzero discord [6]. In a recent survey for discord and some related measures [12], it is mentioned that "Nowadays, there are many ways of understanding the gap in correlations, that is to say the fact that classical correlations and entanglement do not exhaust all possible correlations in quantum systems." And indeed this has also been confirmed by some other tasks like the remote state preparation [5].

In this paper, we examine this notion from the perspective of game theory [15], which studies the situation of two or more players with each having a possibly different goal. There are two broad classes of games. One is strategic-form (or normal-form) games, in which all players make their choice simultaneously; a typical example is Rock-Paper-Scissors. The other class is extensive-form games, in which players make their moves in turn; a typical example is chess.

The research on quantum games began about one decade ago, starting with two pioneering papers.[1] The first one [8] aimed to quantize a specific strategic-form game called *Prisoners' Dilemma* [8], and it unleashed a long sequence of follow-up works in the same model. Despite the rapid growth of literature, controversy also largely exists [3,18,4], which questioned the meaning of the claimed quantum solution, the ad hoc assumptions in the model, and the inconsistency with standard settings of classical strategic games. Recently a new model was proposed for quantizing general strategic-form games [22]. Compared with [8], the new model corresponds to classical games more precisely, and has rich mathematical structures and game-theoretic questions; also see later theoretical developments [11,21,10,17].

Back to the early stage of the development of quantum game theory, the other pioneering paper was [13], which demonstrated the power of using quantum strategies in an extensive-form game. More specifically, Meyer considered the quantum version of the classical Penny Matching game. The basic setting is as follows. There are two players, and each has two possible actions invisible to the opponent on one bit: Flip it or not. Starting with the bit being 0, Player 1 first takes an action, and then Player 2 takes an action, and finally Player 1 takes another action, and the game is finished. If the bit is finally 0, then Player 1 wins; otherwise Player 2 wins. It is not hard to see that if Player 2 flips the bit with half probability, then no matter what Player 1 does, each player wins the game with half probability. Now consider the following change of setting: The bit becomes a qubit; the first player uses a quantum computer in the sense that she can perform any quantum admissible operation on the qubit; the second player uses a classical computer in the sense that she can perform either Identity or the flip operation $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. In this new setting, Player 1 can win the game with certainty! Her winning strategy is simple: she first applies a Hadamard gate to change the state to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, and then no matter whether Player 2 applies the flip operation or not, the state remains the same $|+\rangle$, thus in the third step Player 1 can simply apply a Hadamard gate again to rotate the state back to $|0\rangle$. This shows that a player using a quantum computer can have big advantage over one using a classical computer.

Despite a very interesting phenomenon it exhibits, the quantum advantage is not the most convincing due to a fairness issue. After all, the quantum player takes two actions and the classical player takes just one. And the order of "Player 1 $\to$ Player 2 $\to$ Player 1" is also crucial for the quantum advantage. One remedy is to consider normal-form games, in which the players give their strategies *simultaneously*, thus there is no longer the issue of the action order. Taking the model in [22], two players play a complete-information normal-form game, with a starting state ρ in systems (A_1, A_2), and A_i being given to Player i. A classical player can only measure her part of the state in the computational basis, followed by whatever classical operation C (on the computational basis). In previous works [8,13,23] the classical player is usually assumed to be able to apply any classical operations, followed by a measurement in the computational basis. A classical operation there is implicitly assumed to be unitary, so the operation in the matrix form is a permutation matrix (such as X-gate). Here we allow the classical player to measure first and then perform any classical operation, which gives her more power since the second-step classical operation does not need to be unitary. Indeed, in Meyer's Penny Matching game, in the second step Player 2 could measure the state first and then randomly set it to be $|0\rangle$ or $|1\rangle$ each with half probability. Then in the third step, Player 1's Hadamard gate will change the state to $|+\rangle$ or $|-\rangle$, and in either case, Player 1 could win with only half probability.

Even if we now enlarge the space of possible operations of the classical player, we will show examples where the quantum player has advantage of winning the game. Furthermore, the examples have the following nice properties respecting the fairness of the game:

1. If both players are classical, then both get expected payoff 0, and ρ is a correlated equilibrium in the sense that any classical operation C by one player cannot increase her expected payoff.

[1] Note that there is also a class of "nonlocal games", such as CHSH or GHZ games [2], where all the players have the *same* objective. But general game theory focuses more on situation that the players have *different* objective functions, and the players are selfish, each aiming to optimize her own objective function only.