Information and Computation 256 (2017) 196–211

Contents lists available at ScienceDirect

Information and Computation

www.elsevier.com/locate/yinco

Exact algorithms for Maximum Induced Matching

Mingyu Xiao [a,*], Huan Tan [b]

[a] School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
[b] University of Electronic Science and Technology of China, Chengdu 611731, China

ARTICLE INFO

Article history:
Received 31 July 2015
Available online 11 July 2017

Keywords:
Exact algorithms
Graph algorithms
Maximum induced matching

ABSTRACT

This paper studies exact algorithms for the Maximum Induced Matching problem, in which an n-vertex graph is given and we are asked to find a set of maximum number of edges in the graph such that no pair of edges in the set have a common endpoint or are adjacent by another edge. This problem has applications in many different areas. We give several structural properties of the problem and show that the problem can be solved in $O^*(1.4231^n)$ time and polynomial space or $O^*(1.3752^n)$ time and exponential space.

1. Introduction

Recently, there has been an increasing interest in designing fast and nontrivial exact exponential algorithms for basic NP-hard graph problems. Many interesting exact algorithms have been developed for Maximum Independent Set (MIS) [6,16,23], 3-Coloring [1], Feedback Vertex Set [5,21], Dominating Set [6,10,20], Edge dominating Set [19,22] and many others. Maximum Independent Set is undoubtedly one of the most important problems in exact algorithms. There is a long list of contributions to the running-time bounds of exact algorithms and it can be solved in $O^*(1.1996^n)$ time and polynomial space now [23]. Maximum Independent Set is to find a maximum induced regular graph of degree 0. Gupta, Raman and Saurabh [11] studied exact algorithms for finding maximum induced regular graphs of degree $r \geq 0$ and presented an algorithm with running time $O^*((2-\xi)^n)$, where $0 < \xi < 1$ depends on r. The special case that $r = 1$, i.e., the problem to find a maximum induced regular graph of degree 1, is known as Maximum Induced Matching (MIM). In this paper, we will study structural properties and exact algorithms for Maximum Induced Matching.

To find an *induced matching* (i.e., an induced regular graph of degree 1) of maximum size in a graph has received much attention because of the growing number of applications. Stockmeyer and Vazirani [18] showed that MIM has applications in the risk-free marriage problem – to find a maximum number of married couples such that each married person is compatible with no married person other than his/her spouse. Golumbic and Lewenstein [9] demonstrated some applications of induced matchings in secure communication channels, VLSI design and network flow problems. Golumbic and Laskar [8] gave the relations between the size of a maximum induced matching and the irredundancy number of a graph. MIM is also a subtask of the important problem of finding a strong edge coloring (i.e., a proper coloring of the edges such that no edge is adjacent to two edges of the same color) using a small number of colors (see [4,15] for more information).

It is not surprising that MIM has been extensively studied on computational and algorithmic aspects. Although MIM is polynomial-time solvable in trees [9], chordal graphs [2], circular arc graphs [8], interval graphs [9] and many others, it has been known to be NP-hard in bipartite graphs with maximum degree 4 for more than 30 years [18]. In fact, it remains NP-hard even in planar 3-regular graphs or in planar bipartite graphs with degree-2 vertices in one part and

* Corresponding author.
E-mail addresses: myxiao@gmail.com (M. Xiao), huan1222@gmail.com (H. Tan).

http://dx.doi.org/10.1016/j.ic.2017.07.006
0890-5401/

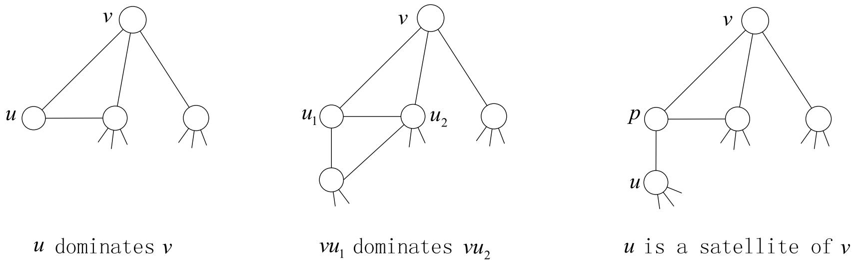

Fig. 1. Illustrations for dominated vertices, dominated edges and satellites.

degree-3 vertices in the other part [4,12]. Kobler and Rotics [13] also showed the NP-hardness of MIM in Hamiltonian graphs, claw-free graphs, chair-free graphs, line graphs and regular graphs.

MIM is hard to approximate or design parameterized algorithms. It is APX-complete even in d-regular graphs for each fixed $d \geq 3$ [4]. There is also an approximation algorithm with asymptotic performance ratio $d-1$ for MIM in d-regular graphs [4]. In general graphs, MIM cannot be approximated within a factor of $n^{1/2-\epsilon}$ in polynomial time for any $\epsilon > 0$ unless $P = NP$ [15]. Take the size k of the induced matching as the parameter. To decide whether there is an induced matching of size at least k is W[1]-hard even in bipartite graphs, but fixed-parameter tractable in planar graphs, line graphs, graphs of bounded treewidth and graphs of girth at least 6 [14].

In terms of exact algorithms for MIM, Gupta, Raman and Saurabh [11] first gave an algorithm with running-time bound $O^*(1.6957^n)$ and then improved it to $O^*(1.4786^n)$. We also note another similar algorithm with the same running-time bound $O^*(1.4786^n)$ [3]. In a previous version of this paper [25], we obtain a running-time bound of $O^*(1.4391^n)$. In this paper, we will improve the running-time bound to $O^*(1.4231^n)$ with polynomial space and $O^*(1.3752^n)$ with exponential space. First, we use an effective method to deal with 'satellites' of vertices with degree ≥ 5 and then we can improve the bottleneck in previous algorithms. Then, we improve the sub-algorithm to compute a maximum induced matching in graphs with maximum degree 4 and get the claimed result of $O^*(1.4231^n)$. Finally, we use the classical dynamic programming to further improve the running-time bound to $O^*(1.3752^n)$. Unlike previous algorithms, our algorithms will not use fast algorithms for Maximum Independent Set as a subalgorithm to deal with graphs with maximum degree 4. Note that in this paper we will use a modified O-notation that suppresses all polynomially bounded factors. For two functions f and g, we write $f(n) = O^*(g(n))$ if $f(n) = g(n) \cdot ply(n)$ for some polynomial function $ply(n)$.

2. Preliminaries

In this paper, a graph always means a simple and undirected graph. Let $G = (V, E)$ be a graph with $n = |V|$ vertices and $m = |E|$ edges. We may simply use v to denote the set $\{v\}$ of a singleton. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. The set of endpoints of edges in an edge set E' is denoted by $V(E')$. For a subgraph (resp., a vertex subset) X, the subgraph induced by $V(X)$ (resp., X) is simply denoted by $G[X]$, and $G[V - V(X)]$ (resp., $G[V - X]$) is also written as $G - X$. For a vertex subset X, let $E(X)$ denote the set of edges between X and $V - X$. Especially, $E(v)$ is the set of edges incident on the vertex v. Let $N(X)$ denote the set of *neighbors* of X, i.e., the vertices $y \in V - X$ adjacent to a vertex $x \in X$, and denote $N(X) \cup X$ by $N[X]$. Let $N_2(v)$ denote the set of vertices with distance exactly 2 from v. The *degree* of a vertex v in a graph G, denoted by $d(v)$, is defined to be the number of neighbors of v in G. A vertex v is *dominated* by a neighbor u of it if v is adjacent to all neighbors of u. An edge v_1v_2 is *dominated* by another edge u_1u_2 if $N[\{v_1, v_2\}] \supseteq N[\{u_1, u_2\}]$. A vertex $u \in N_2(v)$ is called a *satellite* of v if there is a neighbor p of v such that $N[p] - N[v] = \{u\}$. The vertex p is also called the *parent* of the satellite u at v. Fig. 1 gives illustrations for dominated vertices, dominated edges and satellites. A vertex subset V' is called an *independent set* of a graph if there is no edge between any two vertices in V'. An edge subset E' is called an *induced matching* of a graph if the induced graph $G[V(E')]$ has maximum degree 1. We will use $\beta(G)$ to denote the size of a maximum induced matching in G. For dominated vertices and satellites, we will use the following bounds on the number of vertices adjacent to an edge.

Lemma 1. *In a graph without dominated vertices, for any edge vu it holds that*

$$|N[\{v, u\}]| \geq 2 + \max\{d(v), d(u)\}.$$

Proof. Without loss of generality, we assume that $d(v) \geq d(u)$. Note that vertex u has a neighbor w not in $N[v]$, otherwise v would be dominated by u. Then $N[u] - N[v]$ contains at least one vertex w. Since $N[\{v, u\}] = N[v] \cup N[u] = N[v] \cup (N[u] - N[v])$, we know that $|N[\{v, u\}]| = |N[v]| + |N[u] - N[v]| \geq (d(v) + 1) + 1 = d(v) + 2$. □

Lemma 2. *If a vertex v is not dominated by another vertex and has no satellites, then for any edge vu incident on v it holds that*

$$|N[\{v, u\}]| \geq 3 + d(v).$$