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On parametric timed automata and one-counter machines

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ABSTRACT

Two decades ago, Alur, Henzinger, and Vardi introduced the reachability problem for parametric timed automata. Their main results are that reachability is decidable for timed automata with a single parametric clock, and undecidable for timed automata with three or more parametric clocks.

In the case of two parametric clocks, decidability was left open, with hardly any progress that we are aware of in the intervening period.

In this manuscript, we establish a correspondence between reachability in parametric timed automata with at most two parametric clocks and reachability for a certain class of parametric one-counter machines. We leverage this connection (i) to improve decision procedure for one parametric clock from nonelementary to 2NEXP; (ii) to show decidability for two parametric clocks and a single parameter; (iii) to show lower bounds for reachability problem for one and two parametric clocks; (iv) to show decidability for various classes of parametric one-counter machines.

1. Introduction

The problem of reachability in parametric timed automata was introduced over two decades ago in a seminal paper of Alur, Henzinger, and Vardi [2]: given a timed automaton in which some of the constants appearing within guards on transitions are parameters, is there some assignment of integers to the parameters such that an accepting location of the resulting concrete timed automaton becomes reachable?

As is well known, embedded computer systems often depend on the precise timing of individual actions. A standard formalism for modelling such time-dependent systems is that of timed automata—a class of finite automata extended with clocks. In a timed automaton, all clocks evolve simultaneously, at the same rate, and each clock can be reset individually by edges of the automaton. Such edges also usually incorporate timed guards, i.e., simple clock expressions which must be satisfied in order for the corresponding transitions to be enabled, thus constraining the possible evolutions of the timed system.

Getting such timing constraints right is however difficult. Embedded systems further operate in a surrounding environment, and therefore leaving some constraints under-specified in the timed-automata models allows us to elicit the environments in which the system behaves as required. Such considerations led to the introduction of *parametric timed automata* [2] in which some of the constraints feature unspecified parameters. The goal is then to find values of the parameters for which the timed automaton behaves as intended.

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As one might expect, verification of safety properties leads to the reachability problem for parametric timed automata: is a bad state reachable for some values of the parameters? This problem, which is the focus of this manuscript, was first considered by Alur, Henzinger and Vardi [2] over two decades ago, and shown to be undecidable in full generality. More precisely, given a parametric timed automaton, a clock is said to be *nonparametric* if it is never compared with a parameter, and is *parametric* otherwise. Alur et al. showed that, for timed automata with a single parametric clock, reachability is decidable (irrespective of the number of nonparametric clocks). However, the decision procedure given in [2] has provably nonelementary complexity. In addition, Alur et al. showed that reachability becomes undecidable for timed automata with at least three parametric clocks.

The decidability of reachability for parametric timed automata with *two* parametric clocks (and arbitrarily many nonparametric clocks) was left open in [2], with hardly any progress (partial or otherwise) that we are aware of in the intervening period. Alur et al. further showed that this problem subsumes the question of reachability in Ibarra et al.'s "simple programs" [14], also open for over 20 years, as well as the decision problem for a fragment of Presburger arithmetic with divisibility.

In this paper we substantially improve the decision procedure for one parametric clock (from nonelementary to 2NEXP) and give decidability results for classes of two parametric clocks. In this process—and as a main contribution of interest in its own right—we establish a correspondence between reachability in parametric timed automata with at most two parametric clocks and reachability for a certain class of parametric one-counter machines, a formalism which lends itself more conveniently to analysis. In particular, we use this connection to substantially improve Alur et al.'s decision procedure, identify the first nontrivial decidable fragment with two parametric clocks, and make progress on reachability for the simple programs of Ibarra et al.

It is worth noting that similar connections have previously appeared in nonparametric settings [11], and subsequently used to determine the precise complexity of reachability in two-clock (nonparametric) timed automata [7]—another long-standing open problem.

None of our results impose any restriction on the number of nonparametric clocks. More precisely, we show that:

1. The reachability problem for parametric timed automata with a single parametric clock is equivalent to the reachability problem for parametric one-counter machines with updates $-1, 0$ and 1, and comparisons $\leq p_i, \geq p_i$ for parameters p_i.
2. The reachability problem for parametric timed automata with two parametric clocks is equivalent to the reachability problem for parametric one-counter machines with updates $\pm c_i, \pm p_i$, comparisons $\leq p_i, \geq p_i$ and a few other technical operations for constants $c_i \in \mathbb{N}$ and parameters p_i.

In Sec. 7, we then use the relationship between parametric timed automata and the respective classes of parametric one-counter machines to show that:

3. In the case of a single parametric clock (with arbitrarily many nonparametric clocks and arbitrarily many parameters), the reachability problem is in 2NEXP and NEXP-hard improving the nonelementary decision procedure of Alur et al.
4. The reachability problem is decidable for the class of parametric one-counter machines mentioned in (1).
5. The reachability problem is decidable for parametric timed automata with two parametric clocks (and arbitrarily many nonparametric clocks), if the automaton uses only a *single* parameter. Further, the problem is $\mathsf{PSPACE}^{\mathsf{NEXP}}$-hard.
6. We show that the reachability problem is decidable for the class of parametric one-counter machines mentioned in (2) if they use only a single parameter.
7. We use the techniques developed to solve the reachability problem for the simple programs of Ibarra et al. provided they use only a single parameter.

At the time of writing, the result in item 6 corresponds do the largest decidable fragment of parametric timed automata with two parametric clocks. Our decidability results build upon new developments in the theory of one-counter machines [10] and their encodings in Presburger arithmetic [9].

As in [2], our results are presented for timed automata interpreted over discrete time. However, for parametric timed automata with *closed* (i.e., *non-strict*) clock constraints and parameters restricted to ranging over integers,[1] standard digitisation techniques apply [12,23], reducing the reachability problem over dense time to discrete (integer) time.

2. Related work

The decidability of reachability for parametric timed automata has been previously achieved in certain restricted settings. The primary concern in such restricted settings was usually the development of practical verification tools, and indeed the resulting algorithms tend to have comparatively good complexity. However, solutions to these special cases do not lead to general decision problems as is the case in this manuscript. For instance, decidability can be achieved by bounding the

[1] Other researchers have considered variations in which parameters are allowed to range over rationals, yielding different outcomes as regards the decidability of reachability; see, e.g., [21,6], discussed further below.

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