Information and Computation ••• (••••) •••–•••

Contents lists available at ScienceDirect

Information and Computation

www.elsevier.com/locate/yinco

Strongly non-U-shaped language learning results by general techniques

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ARTICLE INFO

Article history:
Received 24 February 2014
Received in revised form 8 June 2015
Available online xxxx

Keywords:
Inductive inference
Non-U-shaped learning
General techniques
Self-learning classes
Infinitary self-referential programs

ABSTRACT

In learning, a semantic or behavioral U-shape occurs when a learner first learns, then unlearns, and, finally, relearns, some target concept.
This paper introduces two general techniques and applies them especially to syntactic U-shapes in learning: one technique to show when they are necessary and one to show when they are unnecessary. The technique for the former is very general and applicable to a much wider range of learning criteria. It employs so-called *self-learning classes of languages* which are shown to *characterize* completely one criterion learning more than another.
We apply these techniques to show that, for set-driven and rearrangement-independent learning, any kind of U-shapes is unnecessary. Furthermore, we show that U-shapes *are* necessary *in a strong way* for iterative learning, contrasting with an earlier result by Case and Moelius that semantic U-shapes are *un*necessary for iterative learning.

1. Introduction

In Section 1.1 we explain U-shaped learning. In Section 1.2 we briefly discuss the general techniques of the present paper and summarize in Section 1.3 our applications of these techniques regarding the *necessity* of U-shaped learning.

1.1. U-shaped learning

U-shaped learning occurs when a learner first learns a correct *behavior*, then abandons that correct behavior and finally returns to it once again. This pattern of learning has been observed by cognitive and developmental psychologists in a variety of child development phenomena, such as language learning [6,31,38], understanding of temperature [38,39], weight conservation [5,38], object permanence [5,38] and face recognition [7]. The case of language acquisition is paradigmatic. For example, a child first uses *spoke*, the correct past tense of the irregular verb *to speak*. Then the child ostensibly overregularizes incorrectly using *speaked*. Lastly and finally the child returns to using *spoke*. The language acquisition case of U-shaped learning behavior has figured prominently in cognitive science [31,34,41].

While the prior cognitive science literature on U-shaped learning was typically concerned with modeling *how* humans achieve U-shaped behavior, the papers [3,11] are motivated by the question of *why* humans exhibit this seemingly inefficient behavior. Is it a mere harmless evolutionary inefficiency or is it *necessary* for full human learning power? A technically

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http://dx.doi.org/10.1016/j.ic.2016.06.015

answerable version of this question is: are there some formal learning classes of tasks for which U-shaped behavior is logically necessary? We first need to describe some formal criteria of successful learning.

An algorithmic learning function h is, in effect, fed an infinite sequence consisting of the elements of a (formal) language L in arbitrary order with possibly some pause symbols # in between elements. During this process, h outputs a corresponding sequence $p(0), p(1), \ldots$ of hypotheses (grammars) which may generate the language L to be learned. A fundamental criterion of successful learning of a language is called *explanatory learning* (TxtEx-*learning*, also called *learning in the limit*) and was introduced by Gold [27]. Explanatory learning requires that the learner's output conjectures stabilize in the limit to a *single* conjecture (grammar/program, description/explanation) that generates the input language. *Behaviorally correct learning* [18,33] requires, for successful learning, convergence in the limit to a *sequence* of correct (but possibly syntactically distinct) conjectures. Another interesting class of criteria features *vacillatory learning* [10,28]. This paradigm involves learning criteria which allow the learner to vacillate in the limit between *at most* some bounded, finite number of syntactically distinct but correct conjectures. For each criterion that we consider above (and below), a *non-U-shaped learner* is naturally modeled as a learner that never returns to a previously *semantically* abandoned correct conjecture on languages it learns according to that criterion.

Ref. [3] showed that every TxtEx-learnable class of languages is TxtEx-learnable by a non-U-shaped learner, that is, for TxtEx-learnability, U-shaped learning is *not* necessary. Furthermore, based on a proof in [24,3] noted that, by contrast, for behaviorally correct learning [23,1,18,33], U-shaped learning *is* necessary for full learning power. In [11] it is shown that, for non-trivial vacillatory learning, U-shaped learning is again necessary (for full learning power). Thus, in many contexts, seemingly inefficient U-shaped learning can actually increase one's learning power.

What turns out to be a variant of non-U-shaped learning is *strongly non-U-shaped* learning essentially defined in [43],[1] where the learner is required never to *syntactically* abandon a correct conjecture on languages it learns according to that criterion. Clearly, *strong* non-U-shaped learnability implies non-U-shaped learnability.[2] In our experience, for theoretical purposes, it is frequently easier to show non-U-shaped learnability by showing *strong* non-U-shaped learnability. Herein we especially study strong non-U-shaped learnability.

1.2. Presented techniques

The present paper presents two general techniques to tackle problems regarding U-shaped learning.

The first general technique can be used to show the *necessity* of U-shapes and employs so-called *self-learning classes of languages*. These are explained in Section 3 below. These self-learning classes of languages provide a (provably) most general way for finding classes of languages that separate two learning criteria, i.e., they give a general way of finding an *example* class of languages learnable with a given learning criterion, but not with another. Theorem 3.6 implies that its presented self-learning classes *necessarily* separate two learnability sets — *iff any class does*. This technique is not specialized only to analyze U-shaped learning, but can be applied to other learning criteria as well. The technique is developed and discussed further in Section 3.

The second general technique is used to show that syntactic U-shapes are *un*necessary and is phrased as a characterization of strongly non-U-shaped learnability of classes of languages (Theorem 4.4).

1.3. Applications of general techniques

A learning machine is *set-driven* [42,37,26,28] (respectively, *rearrangement-independent* [37,26,28]) iff, at any time, its output conjecture depends only on the *set* of non-pause data it has seen (respectively, set of non-pause data *and* data-sequence length), *not* on the *order* of that data's presentation. Child language learning may be insensitive to the order or timing of data presentation; set-drivenness and rearrangement independence, **Sd** and **Ri**, respectively, provide two *local* notions of such insensitivity [10]. It is interesting, then, to see the interaction of these notions with forbidding U-shapes of one kind or another. As we shall see in Section 5, Theorems 5.2 and 5.3, proved with the aid of a general technique from Section 4, imply, for these local data order insensitivity notions, for TxtEx-learning, U-shapes, *even in the strong sense* are *un*necessary.

An *iterative* learner outputs its conjectures only on the basis of its immediately prior conjecture (if any) and its current datum. As we shall see in Section 5, iterative learning provides a (first) example of a setting in which non-U-shaped and strongly non-U-shaped learning are extensionally distinct: [19] shows semantic U-shapes to be *un*necessary for iterative learning, while Theorem 5.7 in the present paper implies that they are *in the strong sense* necessary. To prove this latter result, we actually modify the self-learning class of languages from Theorem 3.6 to make it easier to work with — although the original version *must* work too (by Theorems 3.6 and 5.7).[3]

[1] Wiehagen actually used the term *semantically finite* in place of *strongly non-U-shaped*. However, there is a clear connection between this notion and that of *non-U-shapedness*. Our choice of terminology is meant to expose this connection. See also [21].

[2] For non-U-shaped learning, the learner (on the way to success) must not *semantically* abandon a correct conjecture. In general, semantic change of conjecture is not algorithmically detectable, but syntactic change is. *However*, in the cognitive science lab we can many times see a *behavioral/semantic* change, but it is beyond the current state of the art to see, for example, grammars in people's heads — so we can't *yet* see mere syntactic changes in people's heads.

[3] Some recent papers [13,15,16,30] have also employed (different) self-learning classes for separations.