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Network-formation games with regular objectives [☆]

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ABSTRACT

Classical network-formation games are played on a directed graph. Players have reachability objectives: each player has to select a path from his source to target vertices. Each edge has a cost, shared evenly by the players using it. We introduce and study *network-formation games with regular objectives*. In our setting, the edges are labeled by alphabet letters and the objective of each player is a regular language over the alphabet of labels.
Unlike the case of reachability objectives, here the paths selected by the players need not be simple, thus a player may traverse some edges several times. Edge costs are shared by the players with the share being proportional to the number of times the edge is traversed. We study the existence of a pure Nash equilibrium (NE), the inefficiency of a NE compared to a social-optimum solution, and computational complexity problems in this setting.

1. Introduction

Network design and formation is a fundamental well-studied challenge that involves many interesting combinatorial optimization problems. In practice, network design is often conducted by multiple strategic users whose individual costs are affected by the decisions made by others. Early works on network design focus on analyzing the efficiency and fairness properties associated with different sharing rules (e.g., [24,32]). Following the emergence of the Internet, there has been an explosion of studies employing game-theoretic analysis to explore Internet applications, such as routing in computer networks and network formation [18,1,14,2]. In network-formation games (for a survey, see [38]), the network is modeled by a weighted graph. The weight of an edge indicates the cost of activating the transition it models, which is independent of the number of times the edge is used. Players have reachability objectives, each given by sets of possible source and target nodes. Players share the cost of edges used in order to fulfill their objectives. Since the costs are positive, the runs traversed by the players are simple. Under the common Shapley cost-sharing mechanism, the cost of an edge is shared evenly by the players that use it.

The players are selfish agents who attempt to minimize their own costs, rather than to optimize some global objective. In network-design settings, this would mean that the players selfishly select a path instead of being assigned one by a central authority. The focus in game theory is on the *stable* outcomes of a given setting, or the *equilibrium* points. A Nash equilibrium (NE) is a profile of the players' strategies such that no player can decrease his cost by an unilateral deviation from his current strategy, that is, assuming that the strategies of the other players do not change.[1]

☆ The article is based on the conference publications [5] and [6].
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[1] Throughout this paper, we concentrate on pure strategies and pure deviations, as is the case for the vast literature on cost-sharing games.

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Reachability objectives enable the players to specify possible sources and targets. Often, however, it is desirable to refer also to other properties of the selected paths. For example, in a *communication* setting, edges may belong to different providers, and a user may like to specify requirements like "all edges are operated by the same provider" or "no edge operated by AT&T is followed by an edge operated by Verizon". Edges may also have different quality or security levels (e.g., "noisy channel", "high-bandwidth channel", or "encrypted channel"), and again, users may like to specify their preferences with respect to these properties. In *planning* or in *production systems*, nodes of the network correspond to configurations, and edges correspond to the application of actions. The objectives of the players are sequences of actions that fulfill a certain plan, which is often more involved than just reachability [21]; for example "once the arm is up, do not put it down until the block is placed".

The challenge of reasoning about behaviors has been extensively studied in the context of formal verification. While early research concerned the input-output relations of terminating programs, current research focuses on on-going behaviors of reactive systems [22]. The interaction between the components of a reactive system correspond to a multi-agent game, and indeed in recent years we see an exciting transfer of concepts and ideas between the areas of game theory and formal verification: logics for specifying multi-agent systems [3,11], studies of equilibria in games that correspond to the synthesis problem [10,9,17], an extension of mechanism design to on-going behaviors [27], studies of non-zero-sum games in formal methods [12,8], and more.

In this paper we extend network-formation games to a setting in which the players can specify regular objectives. This involves two changes of the underlying setting: First, the edges in the network are labeled by letters from a designated alphabet. Second, the objective of each player is specified by a *language* over this alphabet. Each player should select a path labeled by a word in his objective language. Thus, if we view the network as a *nondeterministic weighted finite automaton* [15] (WFA, for short) $\mathcal{A}$, then the set of strategies for a player with objective L is the set of accepting runs of $\mathcal{A}$ on some word in L. Accordingly, we refer to our extension as *automaton-formation games*. As in classical network-formation games, players share the cost of edges they use. Unlike the classical game, the runs selected by the players need not be simple, thus a player may traverse some edges several times. Edge costs are shared by the players, with the share being proportional to the number of times the edge is traversed. This latter issue is the main technical difference between automaton-formation and network-formation games, and as we shall see, it is very significant.

Many variants of cost-sharing games have been studied. A generalization of the network-formation game of [2], in which players are weighted and a player's share in an edge cost is proportional to its weight is considered in [13], where it is shown that the weighted game does not necessarily have a pure NE. Resource allocation games [36] are more general and assume there is a *latency function* on each edge that maps the load on the edge to its cost. A special case is congestion games in which the functions are increasing, thus a higher load increases the cost for the players. Studied variants of congestion games include settings in which players' payments depend on the resource they choose to use, the set of players using this resource, or both [31,28,29,20]. In some of these variants a pure NE is guaranteed to exist while in others it is not.

Since a path a player selects in an automaton-formation game need not be simple, a path corresponds to a *multiset* of edges. Thus, automaton-formation games can be viewed as a special case of *multiset resource-allocation games*, where players' strategies consist of multisets of resources. These games are general and subsume previously studied models such as weighted resource-allocation games [29], where each Player i has a weight w_i, and when he selects a subset of resources, he adds a load of w_i on the resources in the selected set. Closer to our multiset games are network routing games in which flow can be split into integral fractions [37] and its generalization to resource-allocation games [23]. There, again each player has a weight, only that he can split the weight between several strategies, assigning an integral weight to each strategy. The relation between automaton-formation games and multiset resource-allocation games is analogue to the relation between network-formation games and resource-allocation games, in the sense that each network-formation game can be viewed as a resource-allocation game, where each simple path in the network corresponds to a subset of the edges. Unlike the case of network-formation games, however, the richness of automaton-formation games make them sufficiently expressive to model every multiset resource-allocation game. Essentially (see Remark 2.1 for the detailed reduction), by associating each resource with a letter from the alphabet, we can translate each strategy in the multiset resource-allocation game to a word that should be traversed in a single-state network in which each resource induces a self-loop. Thus, our results apply also to the (seemingly) more general setting of multiset resource allocation game. Moreover, while the objectives in the resource-allocation game setting are given explicitly, in the automaton-formation game setting they are given symbolically by means of regular languages. This succinctness of the symbolic approach is very significant. In particular, there may be infinitely many strategies to fulfill an objective in an automaton-formation games.

The fact automaton-formation games capture all multiset resource allocation games extends the application of our work. In the context of formal methods, an appealing application of resource-allocation games is that of *synthesis from components*, where the resources are components from a library, and agents need to synthesize their objectives using the components, possibly by a repeated use of some components. In some settings, the components have construction costs (e.g., the money paid to the designer of the component), in which case the corresponding multiset game is a cost-sharing game [4], and our results here can be generalized to apply for this settings. In other settings, the components have congestion effects (e.g., the components are CPUs, and the more players that use them, the slower the performance is), in which case the corresponding game is a multiset congestion game [7].

We study the theoretical and practical aspects of automaton-formation games. In addition to the general game, we consider classes of instances that have to do with the network, the specifications, or to their combination. Recall that the

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