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A fuzzy multi-criteria decision-making model based on simple additive weighting method and relative preference relation

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ABSTRACT

In the past, approaches often generalized classical multi-criteria decision-making (MCDM) methods under fuzzy environment to solve fuzzy multi-criteria decision-making (FMCDM) problems and encompass decision-making messages uncertainty and vagueness. These MCDM methods included analytic hierarchy process (AHP), simple additive weighting method (SAW), technique for order preference by similarity to ideal solution (TOPSIS), etc. In the MCDM methods, SAW is a famous method that is applied in fuzzy environment, but fuzzy multiplication is a drawback to generalize SAW under fuzzy environment. To resolve the multiplication drawback, we utilize a relative preference relation that is from fuzzy preference relation in fuzzy generalized SAW. Generally, fuzzy preference relation is an option to reserve lots of messages. However, pair-wise comparison for fuzzy preference relation is complex on operation. Through the description above, the relative preference relation is improved form fuzzy preference relation to avoid comparing fuzzy numbers on pair-wise and reserving fuzzy messages. Through the relative preference relation, we can generalize SAW under fuzzy environment. It is said that we propose a FMCDM model based on SAW and the relative preference relation to easily and quickly solve FMCDM problems.

1. Introduction

Decision-making is an essential issue for enterprises to find the best alternative from feasible alternatives. Practically, decision-making with several evaluation criteria is multi-criteria decision-making (MCDM) [1–5,8,10,11,15,17–26,29–38,40]. A MCDM model is presented in matrix format as follows.

$$G = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} G_{11} & G_{21} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_m \end{bmatrix} \end{array}$$

and

$$W = [W_1, W_2, \ldots, W_n],$$

where $A_1, A_2, \ldots, A_m$ are feasible alternatives, $C_1, C_2, \ldots, C_n$ are evaluation criteria, G_{ij} is the evaluation rating of A_i on C_i, and W_j is the weight of C_j.

The previous approaches commonly classify MCDM problems into two components. One is classical MCDM problems [3,11,15,19,22,31,40] and the other is fuzzy multi-criteria decision-making (FMCDM) problems (i.e. MCDM problems under fuzzy environment) [1,2,4,5,8,10,17,18,20,21,23–26,29,30,32–38]. In the classical MCDM problems, evaluation ratings and criteria weights are assessed under certain environment and presented by crisp values. On the other hand, evaluation ratings and criteria weights in FMCDM problems are measured on imprecision, subjectivity or vagueness, so that ratings and weights are displayed by linguistic terms [12,16] and then transferred into fuzzy numbers [39,41,42]. Evaluation ratings are indicated by very poor (VP), poor (P), medium poor (MP), fair (F), medium good (MG), good (G) and very good (VG). Criteria weights are denoted as very low (VL), low (L), medium (M), high (H) and very high (VH). The terms above can be converted into fuzzy number. Practically, a problem with several criteria evaluated by linguistic terms is a FMCDM problem.

Through defuzzification or fuzzy generalization, researchers generalized classical MCDM methods under fuzzy environment to solve FMCDM problems. The classical MCDM methods included analytic hierarchy process (AHP) [31], simple additive weighting

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method (SAW) [11], technique for order preference by similarity to ideal solution (TOPSIS) [19], etc. Since defuzzification loses fuzzy messages, fuzzy generalization [5,10,26,30,34–38] is superior to defuzzification on reserving messages. In the classical MCDM methods, SAW proposed by Churchman, Ackoff and Amoff [11] is a well-known method. However, applying SAW under fuzzy environment to solve FMCDM problems [10] was difficult due to fuzzy multiplication. In SAW, evaluation criteria ratings multiplied by corresponding weights are weighted ratings, and the weighted ratings within an alternative are aggregated into the alternative evaluation index. Finally, all alternative evaluation indices are ranked to find the best alternative. In fuzzy generalized SAW, fuzzy multiplication, aggregation and ranking may be critical based on the SAW computations. The complexity and difficulty of fuzzy computations [30,38], especially for fuzzy multiplication, are presented as follows.

In fuzzy generalized SAW, Wei et al. [38], as well as Raj and Kumar [30] multiplied two triangular/trapezoidal fuzzy numbers by extension principle [39,41,42] into a pooled fuzzy number that is not a triangular/trapezoidal fuzzy number. The pooled fuzzy number represents a weighted criterion rating, and weighted ratings within an alternative are aggregated into the alternative evaluation index. Wei et al. [38] used inverse functions to derive the representations of all alternative evaluation indices. On the other hand, Raj and Kumar utilized maximizing and minimizing sets [6] to yield the representations of all alternative evaluation indices. Multiplying triangular fuzzy numbers or trapezoidal fuzzy numbers into a pooled fuzzy number is hard, so are aggregation and ranking [26,30,38] of pooled fuzzy numbers. Therefore, SAW is seldom generalized under fuzzy environment to solve FMCDM problems [10,30,38].

To resolve ties above, we propose a FMCDM model based on SAW and relative preference relation. The relative preference relation is from Lee's [24,25] fuzzy preference relation. In the past, two related SAW approaches including Fan, Ma and Zhang [14], as well as Modarres and Sadi-Nezhad [28] applied preference ratio/information under fuzzy environment. However, the two approaches were unlike our proposed model in practice. Fan et al.'s approach was derived from the approach of Ma et al. [27]. The method yielded fuzzy preference information for MCDM in crisp values, whereas our model's relative preference relation is applied in FMCDM (i.e. MCDM under fuzzy environment). Modarres and Sadi-Nezhad used preference ratio to rank fuzzy numbers prior to any fuzzy arithmetic in the method. On the other hand, our model is based on SAW and relative preference relation to avoid ranking pooled fuzzy numbers. In addition, they utilized area integration to define a fuzzy number's preference function for deriving its preference ratio. The integration application is easy for triangular and trapezoidal fuzzy numbers, but it is difficult for pooled fuzzy numbers. Besides, fuzzy preference relation applied in pooled fuzzy numbers that are formed by fuzzy multiplication and aggregation is a hard work because pooled fuzzy numbers must be compared on pair-wise. Through the utilizing relative preference relation being an improvement of fuzzy preference relation, pooled fuzzy numbers will not be compared on pair-wise. Therefore, we can easily generalize SAW under fuzzy environment to solve FMCDM problems through the relative preference relation.

For the sake of clarity, the rest of this paper is shown as follows. In Section 2, the related concepts of fuzzy theory are presented. The relative preference relation on fuzzy numbers is expressed in Section 3. In Section 4, a FMCDM model that combines SAW with the relative preference relation is displayed. In Section 5, an example is illustrated to describe the FMCDM model clearly. We compare the proposed model with other fuzzy generalized computations in Section 6.

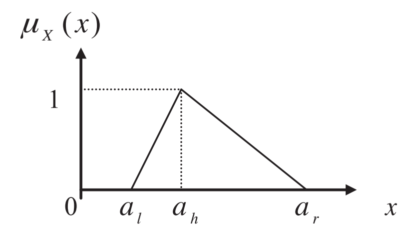

Fig. 1. The membership function of a triangular fuzzy number A.

2. Mathematical preliminaries

In this section, the related definitions of fuzzy sets and fuzzy numbers [39,41,42] are presented below.

Definition 2.1. Let U be a universe set. A fuzzy subset A of U is defined by a membership function $\mu_A(x) \to [0, 1]$, where $\mu_A(x)$, $\forall x \in U$, denotes the degree of x in A.

Definition 2.2 *(:).* The $\alpha - cut$ of fuzzy set A is a crisp set $A_\alpha = \{x \,|\, \mu_A(x) \geq \alpha\}$.

Definition 2.3. The support of fuzzy set A is a crisp set $Supp(A) = \{x \,|\, \mu_A(x) > 0\}$.

Definition 2.4. A fuzzy subset A of U is normal iff $\sup_{x \in U} \mu_A(x) = 1$.

Definition 2.5. A fuzzy subset A of U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, $\forall x,\ y \in U$, $\forall \lambda \in [0,\ 1]$, where $\wedge$ denotes the minimum operator.

Definition 2.6. A fuzzy subset A of U is a fuzzy number iff A is both normal and convex.

Definition 2.7. A triangular fuzzy number A is the fuzzy number with piecewise linear membership function μ_A defined [39,41,42] by

$$\mu_A = \begin{cases} \dfrac{x - a_l}{a_h - a_l}, & a_l \leq x \leq a_h, \\ \dfrac{a_r - x}{a_r - a_h}, & a_h \leq x \leq a_r, \\ 0, & otherwise, \end{cases}$$

which can be indicated as a triplet (a_l, a_h, a_r) in Fig. 1.

Definition 2.8. Let $\circ$ be an operation on real numbers $\Re$, such as $+$, $-$, $*$, $\wedge$ and $\vee$. Let A and B be two fuzzy numbers. By extension principle [39,41,42], an extended operation $\circ$ on fuzzy numbers A and B is defined as

$$\mu_{A \circ B}(z) = \sup_{x,y:z=x \circ y}\{\mu_A(x) \wedge \mu_B(y)\}$$

Definition 2.9. Let A be a fuzzy number. A_α^L and A_α^U are respectively defined as

$$A_\alpha^L = \inf_{\mu_A(z) \geq \alpha}(z)$$

and

$A_\alpha^U = \sup_{\mu_A(z) \geq \alpha}(z)$ [24,25].

Definition 2.10. A fuzzy preference relation R is a fuzzy subset of $\Re \times \Re$ with membership function $\mu_R(A, B)$ representing the preference degree of fuzzy numbers A over B [24,25].

(1) R is reciprocal iff $\mu_R(A, B) = 1 - \mu_R(B, A)$ for all fuzzy numbers A and B.
(2) R is transitive iff $\mu_R(A, B) \geq \frac{1}{2}$ and $\mu_R(B, C) \geq \frac{1}{2} \Rightarrow \mu_R(A, C) \geq \frac{1}{2}$ for all fuzzy numbers A, B and C.
(3) R is fuzzy total ordering iff R is both reciprocal and transitive.