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Conditional edge-fault hamiltonian-connectivity of restricted hypercube-like networks [☆,☆☆]

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ABSTRACT

A graph G is considered conditional k-edge-fault hamiltonian-connected if, after k faulty edges are removed from G, under the assumption that each node is incident to at least three fault-free edges, a hamiltonian path exists between any two distinct nodes in the resulting graph. This paper focuses on the conditional edge-fault hamiltonian-connectivity of a wide class of interconnection networks called restricted hypercube-like networks (*RHLs*). An n-dimensional RHL (RHL_n) is proved to be conditional $(2n-7)$-edge-fault hamiltonian-connected for $n \geq 5$. The technical theorem proposed in this paper is then applied to show that several multiprocessor systems, including n-dimensional crossed cubes, n-dimensional twisted cubes for odd n, n-dimensional locally twisted cubes, n-dimensional generalized twisted cubes, n-dimensional Möbius cubes, and recursive circulants $G(2^n, 4)$ for odd n, are all conditional $(2n-7)$-edge-fault hamiltonian-connected.

1. Introduction

A *graph* $G = (V, E)$ is a pair of the *node set* V and the *edge set* E, where V is a finite set and E is a subset of $\{(u, v)| (u, v)$ is an unordered pair of $V\}$. In addition, $V(G)$ and $E(G)$ are used to denote the node set and edge set of G, respectively. The topological structure of an *interconnection network* (*network* for brevity) can be modeled by a connected graph $G = (V, E)$.

Several conflicting requirements must be considered in designing the topology of interconnection networks. Designing a network topology that is optimal for all scenarios is impossible. Thus, a topology must be designed on the basis of the specific requirements and properties of each network. A central task in designing and evaluating an interconnection network is determining how well other networks can be embedded into the designed network. An *embedding of one guest graph G into*

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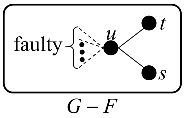

Fig. 1. A worst case scenario that G contains no fault-free hamiltonian path between s and t.

another host graph H is a one-to-one mapping ϕ from the node set of G to the node set of H [23]. An edge in G corresponds to a path in H under ϕ. An embedding strategy provides a scheme for emulating a guest graph on a host graph. A graph embedding problem has two key applications: transplanting parallel algorithms developed for one network to a different network, and allocating concurrent processors to processors in the network.

Paths (*linear arrays*) and *cycles* (*rings*), which are two fundamental networks for parallel and distributed computation, are suitable for designing simple algorithms with low communication costs. Therefore, embedding paths and/or cycles is crucial for a network and has become an area of extensive study.

Since edge (link) faults may occur when a network is activated, it is important to solve problems in faulty networks. The results reported have been obtained by applying two assumptions regarding faulty edges: The first is the *random fault-assumption*, under which no restriction is placed on the distribution of faulty edges. This assumption has been adopted in [10,14,18,29,32]. The second is the *conditional fault-assumption*, which can be classified into two types. Under the first assumption, each node is incident to at least two fault-free edges. This assumption has been adopted in [2,4,7,11,15,17,16,19,22,24,31]. Under the second assumption, each node is incident to at least three fault-free edges. This assumption has been adopted in [6,13].

A *hamiltonian cycle* (*hamiltonian path*) in a graph is a cycle (path) that passes through every node of G exactly once. A graph G is called *hamiltonian* if it contains a hamiltonian cycle, and *hamiltonian-connected* if a hamiltonian path exists between any two distinct nodes in G. When the hamiltonian-connectivity of a graph G is an issue, investigations usually center on whether G is hamiltonian or hamiltonian-connected. Hamiltonian paths and cycles can be applied to practical problems including online optimization of complex, flexible manufacturing systems [1], and wormhole routing [34,35]. These applications motivated this study on embedding networks with hamiltonian paths.

A graph G is considered *conditional k-edge-fault hamiltonian* if, after at most k faulty edges are removed from G, under the assumption that each node is incident to at least two fault-free edges, the resulting graph contains a hamiltonian cycle. Ashir and Stwart [2] showed that the k-ary n-cube is conditional $(4n-5)$-edge-fault hamiltonian. The n-dimensional hypercube is shown to be conditional $(2n-5)$-edge-fault hamiltonian [4,31]. Fu [11] showed that the n-dimensional twisted cube is conditional $(2n-5)$-edge-fault hamiltonian. Furthermore, Hsieh and Wu [17] showed that the n-dimensional locally twisted cube is conditional $(2n-5)$-edge-fault hamiltonian. Hung et al. [19] showed that the n-dimensional crossed cube is conditional $(2n-5)$-edge-fault hamiltonian. Hsieh and Lee [15] investigated the conditional edge-fault hamiltonicity of matching composition networks. More recently, Cheng et al. [7] investigated the conditional edge-fault hamiltonicity of Cartesian product graphs.

In recent years, the conditional fault-assumption has been considered under the assumption that every node is incident to at least two fault-free edges in hamiltonian properties of faulty networks, such as hamiltonian and pancyclic. However, in a hamiltonian-connected graph G with at least four nodes, every node must be incident to at least three fault-free edges for the following reason: consider a graph G with at least four nodes, where u is a node that is incident to n edges in G. Assume that $n-2$ faulty edges are incident to u and only two fault-free edges, (u,s) and (u,t), remain incident to u. In this scenario, a fault-free hamiltonian path between s and t can not exist; that is, $G-F$ is not hamiltonian-connected (Fig. 1). Therefore, it is reasonable to assume that each node in G is incident to at least three fault-free edges.

A graph G is considered *conditional k-edge-fault hamiltonian-connected* if, after k faulty edges are removed from G, under the assumption that each node is incident to at least three fault-free edges, a hamiltonian path exists between any two distinct nodes in the graph. Ho et al. [13] showed that the complete graph with n nodes K_n for $n \geq 4$ and $n \notin \{4,5,8,10\}$ is conditional $(2n-10)$-edge-fault hamiltonian-connected.

This paper addresses the conditional edge-fault hamiltonian-connectivity of a wide class of interconnection networks called *restricted hypercube-like networks* (*RHLs*). An n-dimensional RHL (RHL_n) is proved to be conditional $(2n-7)$-edge-fault hamiltonian-connected for $n \geq 6$. A technical theorem proposed in this paper is then applied to show that several multiprocessor systems, including n-dimensional crossed cubes, n-dimensional twisted cubes for odd n, n-dimensional locally twisted cubes, n-dimensional generalized twisted cubes, n-dimensional Möbius cubes, and recursive circulants $G(2^n,4)$ for odd n, are all conditional $(2n-7)$-edge-fault hamiltonian-connected.

The remainder of this paper is organized as follows: Section 2 introduces the definitions, notation, and properties used in this paper. Section 3 presents an investigation of the conditional edge-fault hamiltonian-connectivities of restricted hypercube-like networks. Section 4 describes applications of the proposed technical theorem to six popular multiprocessor systems. Finally, Section 5 summarizes the conclusion.

2. Preliminaries

Two nodes u and v are *adjacent* if (u,v) is an edge in G. The nodes adjacent to v are called the *neighbors* of v and denoted by $N_G(v)$. The *degree* of node v, denoted by $d_G(v)$, is the number of edges incident to it, that is, $d_G(v) = |N_G(v)|$.