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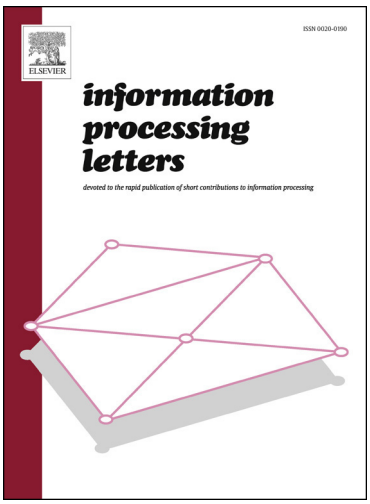

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Oriented, 2-edge-colored, and 2-vertex-colored homomorphisms

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Abstract

Two graph parameters are equivalent if, for every graph class, they are either both bounded or both unbounded. We provide a list of graph parameters equivalent to the acyclic chromatic number, which contains in particular the 2-edge-colored chromatic number. Recently, the CSP dichotomy conjecture has been reduced to the case of 2-edge-colored homomorphism and to the case of 2-vertex-colored homomorphism. Both reductions are rather long and follow the reduction to the case of oriented homomorphism in "Graphs and homomorphisms" by Hell and Nešetřil. We give another proof for the case of 2-vertex-colored homomorphism via a simple reduction from the case of 2-edge-colored homomorphism. Finally, we prove that deciding if the 2-edge-colored chromatic number of a 2-edge-colored graph is at most 4 is NP-complete, even if restricted to 2-connected subcubic bipartite planar graphs with arbitrarily large girth.

1. Introduction

A parameter p defined on simple graphs is *monotone* if $p(G') \leqslant p(G)$ for every subgraph G' of G. Given two monotone parameters p_1 and p_2, we note $p_1 \preceq p_2$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $p_1(G) \leqslant f(p_2(G))$ for every graph G. Moreover, p_1 and p_2 are *equivalent* if $p_1 \preceq p_2$ and $p_2 \preceq p_1$. We note $p_1 \sim p_2$ if p_1 and p_2 are equivalent. For a graph class $\mathcal{G}$ and a monotone parameter p, we define $p(\mathcal{G}) = \max\{p(G) \mid G \in \mathcal{G}\}$. We consider as a graph class any subset of the class of all simple graphs, i.e., it is not necessarily closed under minor, induced subgraph, ... The statement $p_1 \sim p_2$ is equivalent to the statement that for every graph class $\mathcal{G}$, $p_1(\mathcal{G}) = O(1)$ if and only if $p_2(\mathcal{G}) = O(1)$.

The relation $\preceq$ is important in the context of fixed parameter tractability: for every problem P and every parameters p_1 and p_2 such that $p_1 \preceq p_2$, the statement "P is FPT with respect to p_1" is stronger than "P is FPT with respect to p_2".

In section 2, we investigate the family $\mathcal{F}_a$ of graph parameters equivalent to the acyclic chromatic number χ_a. We show that the 2-edge-colored chromatic number χ_2 is in $\mathcal{F}_a$ by showing that χ_2 is equivalent to the oriented chromatic number χ_o, which is in $\mathcal{F}_a$. In Section 3, we show that the dichotomy conjecture for CSP can be reduced from the case of 2-edge-colored homomorphism to the case of 2-vertex-colored homomorphism. In Section 4, we show that 2-edge-colored homomorphism is NP-complete for 2-edge-colored planar graphs with arbitrarily large girth.

2. Bounds on χ_o and χ_2

2.1. Preliminaries

A k-vertex is a vertex of degree k. An oriented graph is obtained from a simple graph by assigning an orientation to every edge. If H_o is an oriented graph, then $A(H_o)$ denotes its set of arcs and $\overrightarrow{uv}$ denotes the arc from the vertex u to the vertex v.