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Multi-criteria group decision making based on trapezoidal intuitionistic fuzzy information

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ABSTRACT

With respect to multi-criteria group decision making (MCGDM) problems under trapezoidal intuitionistic fuzzy environment, a new MCGDM method is investigated. The proposed method can effectively avoid the failure caused by the use of inconsistent decision information and provides a decision-making idea for the case of "the truth be held in minority". It consists of three interrelated modules: weight determining mechanism, group consistency analysis, and ranking and selection procedure. For the first module, distance measures, expected values and arithmetic averaging operator for trapezoidal intuitionistic fuzzy numbers are used to determine the weight values of criteria and decision makers. For the second module, a consistency analysis and correction procedure based on trapezoidal intuitionistic fuzzy weighted averaging operator and OWA operator is developed to reduce the influence of conflicting opinions prior to the ranking process. For the third module, a trapezoidal intuitionistic fuzzy TOPSIS is used for ranking and selection. Then a procedure for the proposed MCGDM method is developed. Finally, a numerical example further illustrates the practicality and efficiency of the proposed method.

1. Introduction

Multi-criteria decision making (MCDM) approach is regarded as a main part of modern decision science and operational research. Due to the complexity of socio-economic environment, multi-criteria group decision making (MCGDM) has become a critical issue recently.

Technique for order preference by similarity to an ideal solution (TOPSIS), developed by Hwang and Yoon [23], is a well-known method for MCDM. In the classical TOPSIS, the decision information can only be expressed with crisp values. However, in the real MCGDM problems, the information available to decision makers are always imprecise and vague. It is unreasonable to express the preferences with crisp values, so the classical TOPSIS method is not applicable for this issue. Furthermore, compared with single MCDM, MCGDM is much more complicated as it consists of multiple experts' subjective judgment and preferences which are often imprecise and vague [36]. Thus, it is necessary to provide some simple and logical mathematical tools for MCGDM under uncertain environment.

Fuzzy set theory proposed by Zadeh [46] can handle vague and imprecise situation, so many researches extend TOPSIS method to fuzzy environment [3,5,9–11,14,15,24–27,29,32,33,35,36,39,42]. Due to the complexity of socio-economic environment, in many practical MCGDM problems, there may hesitation about preferences. In such case, intuitionistic fuzzy set, as an extension of Zadeh's fuzzy set, introduced by Atanassov [4] can be suitable and convenient to express the decision makers' preferences. Recently, intuitionistic fuzzy set has received more and more attention. Accordingly, many researchers have investigated the extended TOPSIS method with intuitionistic fuzzy set such as Boran et al. [8], Park et al. [31], Tan [34] and Ye [44].

However, both the fuzzy set and intuitionistic fuzzy set only use discrete domains. Fuzzy numbers are a special case of fuzzy sets and are of importance for fuzzy MCDM problems [1,2,16,37]. Nehi and Maleki [30] introduced trapezoidal intuitionistic fuzzy numbers as extension of intuitionistic triangular fuzzy numbers. The triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers are the extension of intuitionistic fuzzy sets in another way, which extends discrete set to continuous set [38,45].

Therefore, in our previous works, we try to construct MCGDM methods based on TOPSIS and trapezoidal intuitionistic fuzzy information [12,28]. In the proposed MCGDM methods, the preference information is expressed with trapezoidal intuitionistic fuzzy numbers.

However, the above MCGDM methods based on TOPSIS did not take account of the conflict of decision information. The decision

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makers may be from different fields, so there may be differences in their knowledge structure, preferences, the degree of understanding of the problem and the evaluation ability. What is more, due to the complexity of the problem, there can be significant differences among different decision makers. If there are decision makers whose opinions are significantly different from the group opinion, the aggregated group opinion may come to conclusions which do not meet the reality [20]. The group consistency is directly related to the results of group decision making, thus group consistency analysis research has attracted more and more attention of scholars in recent years, such as García et al. [17]; Herrera et al. [19]; Herrera-Viedma et al. [21,22]; Xu [41]; Ben-Arieh and Easton [6]; Ben-Arieh and Chen [7]. According to the existing achievements, most of the research explores the group consistency problem based on reciprocal judgment matrix, complementary judgment matrix and linguistic judgment matrix. But there is little research on the group consistency analysis based on decision matrix, and, with respect to the form of criteria information, the existing research is based on crisp values, interval numbers and linguistic values, and the consistency analysis in MCGDM problems with trapezoidal intuitionistic fuzzy information does not appear. The calculation of group consistency degree in most studies does not take account of the alternative ranking position which is of great importance in group consistency analysis [20]. Nowadays, the decision makers whose opinions are different from group opinion are required to modify the preference information in lots of consistency analysis algorithms, without taking account of the possibility that the minority may be right.

So the main purpose of this paper is to develop a group consistency analysis procedure based on trapezoidal intuitionistic fuzzy information, and propose a new *MCGDM* model by using consistency analysis as the preceding step of the extended trapezoidal intuitionistic fuzzy TOPSIS method. Specifically, the proposed *MCGDM* model is divided into three interrelated modules: weight determining mechanism, group consistency analysis, and ranking and selection procedure. For the first module, distance measures, expected values and arithmetic averaging operator for trapezoidal intuitionistic fuzzy numbers are used to determine the weight values of criteria and decision makers. For the second module, a consistency analysis and correction procedure based on trapezoidal intuitionistic fuzzy weighted averaging operator and OWA operator is developed to reduce the influence of conflicting opinions prior to the ranking process. For the third module, a trapezoidal intuitionistic fuzzy TOPSIS is used for ranking and selection. Then we develop an algorithm for ranking alternatives under trapezoidal intuitionistic fuzzy environment. Finally, a numerical example further illustrates the practicality and efficiency of the proposed method.

The rest of the paper is organized as follows. In Section 2, we firstly introduced some basic notations and preliminary definition of trapezoidal intuitionistic fuzzy numbers. In Section 3, a new MCGDM model based on trapezoidal intuitionistic fuzzy information is proposed. In Section 4, an illustrative example shows the feasibility and availability of the proposed method. The paper is concluded in Section 5.

2. Preliminaries

Some basic definitions of intuitionistic fuzzy sets [4], fuzzy numbers [16], intuitionistic fuzzy numbers [18] and trapezoidal intuitionistic fuzzy numbers [30] are introduced in this section.

Definition 1 *([4])*. Let a set X be fixed. An intuitionistic fuzzy set A in X is defined as:

$$A = \{\langle X, \mu_A(x), \nu_A(x) | x \in X\rangle\}, \quad (1)$$

where the functions $\mu_A(x) : X \to [0, 1]$, $\nu_A(x) : X \to [0, 1]$ are the degree of membership and non-membership of the element $x \in X$, respectively, satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$. We call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ the intuitionistic fuzzy index of x *to* A, which represents the degree of hesitancy of x.

Definition 2 *([16])*. Let A be a fuzzy number on a real number set R, its membership is defined as follows:

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ f_A(x), & a_1 \le x < a_2 \\ 1, & a_2 \le x \le a_3 \\ g_A(x), & a_3 < x \le a_4 \\ 0 & x > a_4 \end{cases} \quad (2)$$

where $a_1, a_2, a_3, a_4 \in R$, $f_A(x) : X \to [0, 1], g_A(x) : X \to [0, 1]$ are called the left side and the right side of a fuzzy number A, respectively. And f_A is nondecreasing continuous function, g_A is nonincreasing continuous function, respectively.

Definition 3 *([18])*. Let A be an intuitionistic fuzzy number on a real number set R, its membership and non-membership are defined as follows:

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ f_A(x), & a_1 \le x < a_2 \\ 1, & a_2 \le x \le a_3 \\ g_A(x), & a_3 < x \le a_4 \\ 0 & x > a_4 \end{cases} \quad (3)$$

$$\nu_A(x) = \begin{cases} 1 & x < b_1 \\ h_A(x), & b_1 \le x < b_2 \\ 0, & b_2 \le x \le b_3 \\ k_A(x), & b_3 < x \le b_4 \\ 1 & x > b_4 \end{cases} \quad (4)$$

where $0 \le \mu_A(x) \le 1, 0 \le \nu_A(x) \le 1, \mu_A(x) + \nu_A(x) \le 1$. $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$, such that $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and four functions $f_A, g_A, h_A, k_A : R \to [0, 1]$ are called the sides of a fuzzy number. And f_A, k_A are nondecreasing continuous function, g_A, h_A, are nonincreasing continuous function.

The α-cuts provide a useful tool for dealing with fuzzy numbers. As for intuitionistic fuzzy number, it is necessary to distinguish following α-cuts: $(A^+)_\alpha$ and $(A^-)_\alpha$.

Definition 4 *([30])*. The α-cuts of the intuitionistic fuzzy number A on a real number set R is defined as follows:

$$(A^+)_\alpha = \{x \in R | \mu_A(x) \ge \alpha\} \quad (5)$$

$$(A^-)_\alpha = \{x \in R | 1 - \nu_A(x) \ge \alpha\} \quad (6)$$

According to Definition 4, every α-cut is a closed interval. Hence, we have $(A^+)_\alpha = [A_L^+(\alpha), A_U^+(\alpha)]$ and $(A^-)_\alpha = [A_L^-(\alpha), A_U^-(\alpha)]$, respectively, where

$$A_L^+(\alpha) = \inf(x \in R | \mu_A(x) \ge \alpha) \quad (7)$$

$$A_U^+(\alpha) = \sup(x \in R | \mu_A(x) \ge \alpha) \quad (8)$$

$$A_L^-(\alpha) = \inf\{x \in R | 1 - \nu_A(x) \ge \alpha\} \quad (9)$$

$$A_U^-(\alpha) = \sup\{x \in R | 1 - \nu_A(x) \ge \alpha\} \quad (10)$$

Definition 5 *([30])*. Let A be a trapezoidal intuitionistic fuzzy number with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and