



Morpion Solitaire 5D: A new upper bound of 121 on the maximum score ☆,☆☆



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ABSTRACT

Morpion Solitaire is a pencil-and-paper game for a single player. A move in this game consists of putting a cross at a lattice point and then drawing a line segment that passes through exactly five consecutive crosses. The objective is to make as many moves as possible, starting from a standard initial configuration of crosses. For one of the variants of this game, called 5D, we prove an upper bound of 121 on the number of moves. This is done by introducing *line-based analysis*, and improves the known upper bound of 138 obtained by potential-based analysis.

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1. Introduction

Morpion Solitaire, also known as *Join Five*, is a game played alone with a pencil and paper, and it is popular in several countries [4]. A move in this game consists of drawing a cross and a line segment on an infinite square lattice. The line segment has to pass through exactly five consecutive crosses including the one that has just been placed. The objective is to make as many moves as possible starting from a given initial configuration. We call the number of moves the *score*. There are two variants of this

game according to how two line segments can touch each other.

Demaine et al. [6] studied generalizations of the game and their computational complexity, and show that a generalized *Morpion Solitaire* is NP-hard and that its maximum score is hard to approximate. Another target of interest is the maximum scores or their lower and upper bounds. Recently, computing maximum scores was used as a test problem to evaluate the effectiveness of the Monte-Carlo tree search method, which has been attracting rising attention as a promising approach in game programming [5,9].

In this paper, we focus on the 5D variant of the game, and show improved upper bounds on the maximum score. We first show that the known upper bound of 138 can be improved to 136 by pushing on the existing potential-based approach. Next we introduce a line-based approach and further improve the bound to 121. We also try to organize and present related results, since there are relatively few research papers on this topic.

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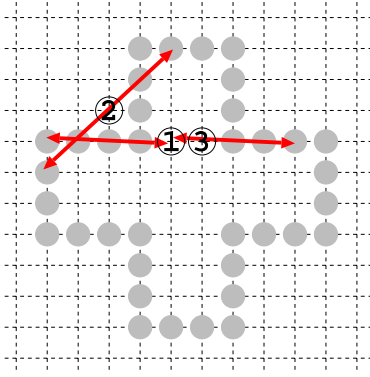


Fig. 1. The standard initial board layout for Morpion Solitaire 5D and 5T, and an example of the first three moves. Each cross placed in these moves is denoted by a number surrounded by a circle. Move 3 is allowed in 5T (touching) but not in 5D (disjoint).

2. Rules and records

2.1. Rules

Morpion Solitaire is played on an infinite square lattice. Initially 36 crosses are drawn on lattice points so that they form a large cross shape with edge length 4 as shown in Fig. 1. In this figure, a cross is denoted by a circle. (In this paper, the length of a line segment means the number of crosses covered by it.)

A *move* consists of the following two steps applied in this order. The objective of this game is to maximize the number of moves.

- (1) Draw a new cross on a lattice point which is empty (no cross exists) on the current board.
- (2) Draw a segment of length 5 (called a *line*) that passes through exactly five consecutive crosses including the one drawn in step 1 of this move. Here, the line can be drawn in either one of the four directions, vertical, horizontal, or diagonal. Two lines in the same direction may not overlap.

There are two variants of this game depending on whether two lines in the same direction can touch (5T) or have to be disjoint (5D) (Fig. 1). We mainly discuss about 5D in this paper.

When a line L passes a cross C , we say that L covers the cross or the lattice point on which it is drawn. We sometimes call a board after move N a board at move N . Also we sometimes denote a cross and a line drawn in move N by C_N and L_N , respectively.

2.2. Records

The above definition of the game can be extended to α D and α T, where the lines have length α and the edges of the large cross in the initial configuration have length $\alpha - 1$, however, the maximum scores are known for all variants except $\alpha = 5$. For 3T and 3D, the maximum scores are not bounded, as there are sequences of moves that can be repeated infinitely [6]. For 6T and 6D, we can easily see

Table 1

Records on Morpion Solitaire 5T and 5D: their maximum achieved scores and proven upper bounds.

Game type	Best achieved score	Upper bound
5T	178	705
5D	82	138

that the maximum score is 12. For 4T and 4D, there used to be gaps between the maximum achieved scores and the upper bounds in the past, but in 2007, 62 and 35 moves were achieved for 4T and 4D, respectively [7], and these scores were proved to be optimal in 2008 [4].

Table 1 [4] shows the current maximum scores of 5T and 5D. We briefly explain how the records of these two variants have been developed.

5T. Bruneau achieved 170 in 1976 by hand [2]. In 2010, by computer, Akiyama, Komiya and Kotani [1] used Monte-Carlo tree search to achieve 145 and 146, which were still less than human's record at that time. From 2010 to 2011, also by computer, Rosin achieved 172, beating human's record [3]. Rosin [9] improved the record to 177 in 2011, and the current record is 178 [10]. An upper bound of 705 on the maximum score is known [6].

5D. According to Demaine et al. [6], 68 moves was achieved by hand in 1999. Cazenave [5] established 80 in 2008, and then Rosin [9] improved it to 82 in 2010, both by computers. As for upper bounds, Demaine et al. [6] showed 141 in 2006 [6] and Karjalainen showed 138 in 2011 [8].

Recent records of maximum scores of both 5T and 5D were obtained by computers. The framework used for this was Monte-Carlo tree search or its extensions, which are known to produce excellent results in designing computer programs, for example, for playing Shogi or Go against humans.

Hereafter, in this paper, we focus only on 5D variant and aim to improve the upper bound on its maximum score, which is known to be 138.

3. Potential-based analysis of upper bounds

The known upper bound of 138 on the maximum score of Morpion Solitaire 5D is obtained by arguments using 'potentials'. In this section, we explain potentials and the related results, and then show that the upper bound can be improved to 136 by a more detailed analysis based on this approach.

3.1. Preceding research

The notion of potential in the analysis of Morpion Solitaire seems to have been originally introduced in folklore discussions and was used by Demaine et al. [6]. The *potential* of a cross on a board is the number of additional lines that can cover it. Since a cross can be covered by at most four lines (in the vertical, horizontal and two diagonal directions), the potential of a cross C is formally given by

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