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A note on path embedding in crossed cubes with faulty vertices ☆

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ABSTRACT

In this note, we investigate the problem of embedding paths of various lengths into crossed cubes with faulty vertices. In Park et al. (2007) [14] showed that, for any hypercube-like interconnection network of 2^n vertices with a set F of faulty vertices and/or edges, there exists a fault-free path of length ℓ between any two distinct fault-free vertices for each integer ℓ satisfying $2n-3 \leqslant \ell \leqslant 2^n-|F|-1$. In this note, we show that, for crossed cubes CQ_n with $n \geqslant 5$, the range of ℓ can be extended to $[2n-5, 2^n-|F|-1]$. Moreover, we also show that the vertices of CQ_5 can be partitioned into two symmetric groups.

1. Introduction

A *path* P from vertex x to vertex y in a graph $G=(V,E)$ is a sequence of distinct vertices $v_1, v_2, \dots, v_{\ell+1}$ with $v_1=x$, $v_{\ell+1}=y$, and $v_iv_{i+1} \in E(G)$ for $1 \leqslant i \leqslant \ell$. We use $\langle v_1, v_2, \dots, v_{\ell+1}\rangle$ to denote such a path. The *length* of a path P is the number of edges in P. If a path P is of length ℓ, then there are $\ell+1$ vertices in P and we use $P_{\ell+1}$ to denote the path. The problem of embedding paths of various lengths into a hypercube-like interconnection network has been extensively studied [3–6,8,9,11,12,14,15]. If there exists a set F of faulty vertices, then the path embedding problem is to find out fault-free paths of each length. The results on embedding paths of various lengths into crossed cube CQ_n, which will be defined in Section 2, are listed in Table 1, where $|F|$ is the number of faulty vertices and $d(x,y)$ is the distance between vertices x and y.

In this note, we investigate the problem of embedding fault-free paths of various lengths into a crossed cube with faulty vertices. We show that there exists a fault-free path of length ℓ between any two distinct fault-free vertices for each integer ℓ satisfying $2n-5 \leqslant \ell \leqslant 2^n-|F|-1$, where F is the set of faulty vertices in CQ_n with $n \geqslant 5$ and $|F| \leqslant n-3$. This improves the results in [14] in which Park, Lim, and Kim showed that there exists a fault-free path of each length from $2n-3$ to $2^n-|F|-1$ in a hypercube-like interconnection network.

The rest of this note is organized as follows. In Section 2, the formal definition and some properties of crossed cubes are introduced. In Section 3, we show that there are two symmetric groups in CQ_5 as well as the main result of this note. Finally, concluding remarks are given in Section 4.

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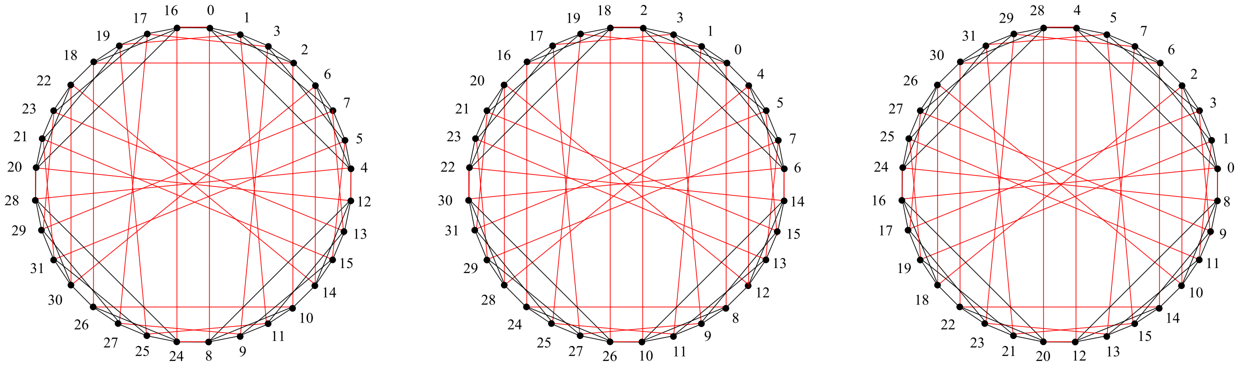
Table 1
Related results on embedding paths into CQ_n.

Reference No.	The range of n	$\|F\|$	The smallest length	The longest length
[3]	$n \geqslant 3$	0	$\lceil \frac{n+1}{2} \rceil + 1$	$2^n - 1$
[4,16]	$n \geqslant 3$	0	$d(x, y) + 2$	$2^n - 1$
[13]	$n \geqslant 3$	$n - 3$	$2^{n-1} + 1$	$2^n - \|F\| - 1$
[14]	$n \geqslant 3$	$n - 3$	$2n - 3$	$2^n - \|F\| - 1$
This note	$n \geqslant 5$	$n - 3$	$2n - 5$	$2^n - \|F\| - 1$
Our conjecture	$n \geqslant 2k + 1$, $k \geqslant 1$	$n - 3$	$2(n - k) - 1$	$2^n - \|F\| - 1$



(a) CQ_5 (b) $\phi(0) = 2$ and $\phi(4) = 6$ (c) $\phi(0) = 4$

Fig. 1. Automorphisms of CQ_5.

2. Preliminaries

Crossed cubes were introduced by Efe in [2]. An n-dimensional crossed cube CQ_n contains 2^n vertices in which the degree of every vertex is n. Every vertex of CQ_n is identified by a unique binary string of n bits which is also called an *address*. An n-dimensional crossed cube is defined recursively as follows.

Definition 2.1. Two 2-bit binary strings x_2x_1 and y_2y_1 are pair related, denoted by $x_2x_1 \sim y_2y_1$, if and only if $(x, y) \in \{(00, 00), (10, 10), (01, 11), (11, 01)\}$.

Definition 2.2. CQ_1 is a complete graph of two vertices with addresses 0 and 1, respectively. Crossed cube CQ_n, for $n \geqslant 2$, consists of two subcubes CQ^0_{n-1} and CQ^1_{n-1} in which CQ^0_{n-1} contains all vertices with addresses $u = u_{n-1}u_{n-2}\dots u_0$ and CQ^1_{n-1} contains all vertices $v = v_{n-1}v_{n-2}\dots v_0$, where $u_{n-1} = 0$ and $v_{n-1} = 1$. Two vertices u and v are joined by an edge in CQ_n if and only if

(1) $u_{n-2} = v_{n-2}$ when n is even, and
(2) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$ for all i with $0 \leqslant i \leqslant \lfloor \frac{n-1}{2} \rfloor$.

Let $CQ^y_{n-|y|}$ denote a subcube of CQ_n in which all vertices have prefix y in their addresses and $|y|$ is the number of bits in y. For a vertex $u \in CQ_n$, the *open neighborhood* of u is the set $N(u) = \{v \in V(CQ_n) | uv \in E(CQ_n)\}$. Let $N_{CQ^y_{n-|y|}}(u)$ denote the set of vertices in $N(u) \cap V(CQ^y_{n-|y|})$. For simplicity, when $N_{CQ^y_{n-|y|}}(u)$ contains exactly one vertex, we also use $N_{CQ^y_{n-|y|}}(u)$ to denote the vertex in the set. For example, see Fig. 1(a) which depicts CQ_5. The subcube CQ^{00}_3 contains vertices $0, 1, \dots, 7$ as all of these have 00 as the prefix in their addresses. The neighbors of vertex 0 are the vertices in $N_{CQ_5}(0) = \{1, 2, 4, 8, 16\}$. Note that $N_{CQ^{01}_3}(0) = 8$ and $N_{CQ^1_4}(0) = N_{CQ^{10}_3}(0) = 16$.

Definition 2.3. A path (respectively, cycle) is called a *Hamiltonian path* (respectively, *Hamiltonian cycle*) of G if it spans G. A graph G is *Hamiltonian* if it has a Hamiltonian cycle, and is called *Hamiltonian connected* if it contains a Hamiltonian path between any pair of distinct vertices. A graph G is *f-fault Hamiltonian* (respectively, *f-fault Hamiltonian connected*) if it remains Hamiltonian (respectively, Hamiltonian connected) after removing at most f vertices and/or edges.

Let F be a set of faulty vertices. We use $G - F$ to denote the resulting graph after the vertices in F are removed from G.

Theorem 2.4 *([3]). For $n \geqslant 3$, there exists a path of length ℓ between any two distinct vertices x and y in CQ_n, where ℓ is an integer satisfying $\lceil \frac{n+1}{2} \rceil + 1 \leqslant \ell \leqslant 2^n - 1$.*

Theorem 2.5 *([7]). For $n \geqslant 3$, crossed cube CQ_n is $(n-2)$-fault Hamiltonian and $(n-3)$-fault Hamiltonian connected.*

Theorem 2.6 *([14]). In a restricted hypercube-like graph G of degree $n \geqslant 3$, each pair of vertices are joined by a path in $G - F$ of each length from $2n - 3$ to $|V(G - F)| - 1$ for any set F of faulty elements (vertices and/or edges) with $|F| \leqslant n - 3$.*