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Diagnosable evaluation of enhanced optical transpose interconnection system networks

Chang-Hsiung Tsai *, Jheng-Cheng Chen

Department of Computer Science and Information Engineering, National Dong Hwa University, Hualien, 970 Taiwan, ROC

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ABSTRACT

The process of identifying faulty processors is called diagnosis of the system. Several models of diagnosis have been proposed, the most popular being the PMC (Preparata, Metze and Chien) diagnostic model proposed by Preparata et al. in 1967. The precise strategy correctly identifies all faulty processors while the pessimistic strategy isolates all faulty processors within a set containing at most one fault-free processor. For a multiprocessor system, diagnosability is critical to measure its performance. The enhanced optical transpose interconnection system (enhanced OTIS), network has important applications in parallel processing. In this network architecture, n^2 processors are divided into n groups of n processors; processors in the same group are connected by electronic links while the groups are simultaneously connected by optical links. An enhanced OTIS network is regular if its base graph G is regular. In this paper, we discuss fault diagnosis in an enhanced OTIS network, including both the precise strategy and pessimistic strategy under the PMC diagnostic model.

1. Introduction

Even though improvement in technology has made it possible to realize very powerful multiprocessor systems, the bandwidth limitations imposed by electronic interconnections prove to be a major bottleneck. Optical interconnection networks would be better candidates than conventional interconnection networks for multiprocessor systems. A popular realization of optical communication is the Optical Transpose Interconnection System (OTIS) [11]. OTIS networks have a base graph G, on n vertices, and consist of n disjoint copies of G. These copies are labelled $G_1, G_2, \ldots, G_n$ and the vertices of any copy are $v_1, v_2, \ldots, v_n$. The edges involved in any one of these copies of G are intended to model electronic connections whereas additional edges, from vertex v_i of copy G_j to vertex v_j of copy G_i, for every $i, j \in \{1, 2, \ldots, n\}$, with $i \neq j$, are intended to model optical connections. The resulting OTIS network is denoted $OTIS(G)$. One slightly displeasing aspect of OTIS networks is that no matter what the base graph G is, the corresponding OTIS network $OTIS(G)$ cannot be regular. We note that there is no optical link from the node v_i of G_i for any copy of G_i in $OTIS(G)$. Several studies of OTIS architecture have been proposed [2,14,15]. In order to improve the regularity of OTIS networks, enhanced OTIS networks were proposed by Das in 2007 [4]. Enhanced OTIS networks retain the corresponding OTIS as a subgraph and thus have almost all the desirable properties of the corresponding OTIS but contain additional optical links. Several advantages are gained by adding those extra links, namely uniform node degree, increased diagnosability, and improved fault-tolerance. These advantages make enhanced OTIS a suitable architecture for multiprocessor interconnection networks.

The process of identifying faulty processors is called diagnosis of the system. The PMC diagnostic model proposed

* Corresponding author.
E-mail address: chtsai@mail.ndhu.edu.tw (C.-H. Tsai).

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by Preparata, Metze and Chien [17] is the most popular diagnostic model. In this model, every processor performs tests on its neighbors based on the communication links between them. When one processor tests another, the tester declares the tested processor to be fault-free or faulty depending on the test response; the result is always accurate if the tester is fault-free, but if the tester is faulty, the result is unreliable. According to the traditional *precise diagnosis* strategy, all processors must be identified correctly. A system is *t-diagnosable* if all faulty processors can be identified under the precise strategy provided that the number of faults presented does not exceed t. The maximum number of faulty processors that the system can guarantee to identify is called the diagnosability of the system. The *pessimistic diagnosis* strategy proposed by Kavianpour and Friedman [8] is a process of diagnosing faults that allows all faulty processors to be isolated within a set having at most one fault-free processor. A system is t_1/t_1-diagnosable if, provided the number of faulty processors is bounded by t_1, all faulty processors can be isolated within a set of size at most t_1 with at most one fault-free node mistaken as a faulty one. Numerous studies have been dedicated to the PMC model [3,5,7,12,13,16,18,19].

In this paper, we show that an enhanced OTIS network with base graph G which has even number of nodes and degree $t \geq 2$ is $(t+1)$-diagnosable, and $(t_1+2)/(t_1+2)$-diagnosable if G is a t_1/t_1-diagnosable regular graph. Our paper is organized as follows: In Section 2, we introduce graph theory terminology and fundamental properties. In Section 3, the precise diagnosability and pessimistic diagnosability of enhanced optical transpose interconnection system networks under the PMC model are evaluated. Finally, we give a conclusion in Section 4.

2. Preliminaries

The underlying topology of a multiprocessor system is usually represented by a graph $G = (V, E)$, where each of the vertices $u \in V$ denotes a processor and each edge $(u, v) \in E$ denotes an undirected communication link from processor u to processor v. Throughout this paper, three terms – network, system, and graph – will be used interchangeably. A loop edge is an edge for which the two endpoints are the same vertex. Two edges are multiple edges if they have exactly the same two endpoints. A graph is simple if it does not contain loop edges or multiple edges. In this paper, we focus on simple graphs. For graph theory terminology and notation not defined here, we follow [9].

Let $G = (V, E)$ be a graph. For a vertex $u \in V(G)$, we define $N_G(u)$ to be the set of vertices adjacent to u. The size of $N_G(u)$ is called the degree of u denoted by $deg_G(u)$. In addition, the degree of a graph G is defined as $\delta(G) = min_{u\in V} deg_G(u)$. For a vertex set $U \subseteq V(G)$, let $N_G(U) = \bigcup_{u\in U} N_G(u) \setminus U$. If $|N_G(u)| = k$ for any vertex u in G, then G is called a k-regular graph. A graph $H = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. The components of a graph G are its maximal connected subgraphs. A component is trivial if it has no edges; otherwise, it is nontrivial. In addition, for a vertex set $S \subseteq V$, the notation $G - S$ means the graph obtained by deleting all the vertices in S from G and all edges in G connected to S.

Optical transmissions, owing to its inherent parallelism, high spectral and spatial bandwidth, low latency and signal crosstalk, reduced power consumption and desirable topological properties [6], possesses the potential to be a better solution to several communication problems in parallel and distributed computing. Optical interconnection networks would be a better candidate than conventional interconnection networks for multiprocessor systems. A popular realization of optical communication is the Optical Transpose Interconnection System (OTIS) [11]. The OTIS(G) network has a base graph G with n vertices and consists of n copies of G, namely $G_1, G_2, \ldots, G_n$. Let the vertices of any copy be $v_1, v_2, \ldots, v_n$ and add an edge connecting v_i of copy G_j to v_j of G_i for $1 \leq i, j \leq n$ with $i \neq j$. The formal definition of $OTIS(G)$ is as follows.

Definition 1. Let $G = (V, E)$ be an undirected graph. The $OTIS(G)$ graph is an undirected graph given by:

$$V(OTIS(G)) = \{\langle v_g, v_p\rangle | v_g, v_p \in V(G)\} \text{ and}$$

$$\begin{aligned} E(OTIS(G)) &= \{(\langle v_g, v_r\rangle, \langle v_g, v_s\rangle) | v_g \in V(G), (v_r, v_s) \in E(G)\} \\ &\quad \cup \{(\langle v_g, v_p\rangle, \langle v_p, v_g\rangle) | v_g, v_p \in V(G), \text{ and } v_g \neq v_p\}. \end{aligned}$$

Obviously, $OTIS(G)$ is not regular no matter what the base graph G is. Therefore, in order to solve the problem of regularity, a variation of $OTIS(G)$ called *enhanced* $OTIS(G)$ was proposed in [4]. Given a base graph G with vertices set $\{v_1, v_2, \ldots, v_n\}$, the enhanced $OTIS(G)$ denoted by $EOTIS(G)$ is obtained from $OTIS(G)$ by adding an edge from node $\langle v_g, v_g\rangle$ to $\langle v_{\bar{g}}, v_{\bar{g}}\rangle$, where $1 \leq g \leq n$ and $\bar{g} = n - g + 1$. For example, graphs $OTIS(C_4)$ and $EOTIS(C_4)$ are illustrated in Fig. 1. Obviously, $EOTIS(G)$ includes self-loop edges if G has odd number of nodes. Hence in this paper we will only consider the base graph G with an even number of nodes.

Therefore, $EOTIS(G)$ is regular if its base graph G is regular, and the following lemma is obviously obtained.

Lemma 1. $\delta(EOTIS(G)) = \delta(G) + 1$.

Many diagnostic models of system-level diagnosis have been proposed in the literature [1,10,17]. Among these models, the most popular is the PMC diagnostic model (or PMC model for short) proposed by Preparata, Metze, and Chien [17]. Under the PMC model, a self-diagnosable system G was often modelled by a digraph in which an arc direct from vertex u to vertex v means that u can test v. In this situation, u is a tester and v is a tested vertex. The edge orientation is based on unit type (test or tested unit). The directed edge (u, v), with binary weight $\sigma(u, v)$, exists if and only if units u and v are interconnected and unit v (tested unit) is tested by unit u with binary outcome $\sigma(u, v)$. The test outcome $\sigma(u, v)$ is 0 (1) if u diagnoses v as fault-free (faulty). If u is faulty, then the outcome $\sigma(u, v)$ is unreliable. The collection of all test outcomes is called a *syndrome*.