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An improved time-dependent Boundary Element Method for two-dimensional acoustic problems in a subsonic uniform flow

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A B S T R A C T

In this paper, we have developed an improved formulation of two-dimensional Convected Boundary Element Method (CBEM) for radiation and propagation acoustic problems in a uniform mean flow with arbitrary orientation. The improved CBEM approach is derived from an advanced form of time-space two-dimensional Boundary Integral Equation (BIE) according to new Sommerfeld Radiation Conditions (SRC) with arbitrary mean flow. The acoustic variables of these formulations are expressed only in terms of the acoustic field as well as its normal and tangential derivatives. The multiplication operators are based explicitly on the two-dimensional Green's function and its convected normal derivative kernel. The proposed terms significantly reduce the presence of flow quantities incorporated in the classical integral formulations. Precisely, the convected kernel only requires the evaluation of two terms instead of several terms in conventional formulations due to the flow effects in the temporal and spatial derivatives. Also, for the singular integrations, the kernels containing logarithmic and weak singularities are converted to regular forms and evaluated partially analytically and numerically. The formulation is derived to be easy to implement as a numerical tool for computational codes of acoustic mediums with arbitrary mean flow. The accuracy and robustness of this technique is assessed through several examples such as the two-dimensional monopole, dipole and quadrupole sources in a uniform mean flow. An application of the improved formulation coupled with Particular Dirichlet-to-Neumann operators (PDtN) has been presented to describe the acoustic field inside two-dimensional infinite ducts in a uniform mean flow. The numerical results are compared to analytical, conventional BEM and Finite Element Method (FEM) formulations.

1. Introduction

Noise pollution in many industrial applications such as airplanes, satellites and automobiles devices, can be predicted by some techniques focusing on acoustic radiation and propagation of sound waves into the far-field, which is derived by the noise sources in a mean flow. In particular, the sound radiation through a turbofan engine of the aircraft is caused by the noise sources in an air flow [1]. In this case, several theoretical models and numerical schemes are developed to reduce the noise effects and to model any type of acoustic problems [2]. All analytical models are adapted to resolve uniform acoustic configurations such as the two-dimensional and axisymmetric ducts with rigid and absorbent walls [3–5]. Whereas, the main numerical methods used to simulate complex acoustic problems are the Finite Element Method (FEM) which is based on the variational formulations [6–8], and the Boundary Element Method (BEM) which is derived from the integral representations [9–11]. Precisely, the finite element method is more adapted for bounded mediums otherwise the Boundary Element Method is generally used to solve the radiation acoustic mediums in a free space. A large class of the complex problems due to the boundary conditions requires a coupled BEM/FEM formulation [12] in which a continuous solution is obtained by matching the finite and the integral element solutions at the interface between two regions. Also, the BE and FE methods can be developed in frequency and time mediums [13,14].

The boundary integral formulation has main advantages over the finite element method in bounded and infinite mediums with mean flow. The most important advantages of BEM are respectively the CPU time, the radiation condition, and the mesh method. Firstly, the

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Boundary Element Method reduces the original dimensions by one compared to the finite element method, which leads to decreases the computation and implementation expenses [15]. Secondly, it is known that the integral representation satisfies the radiation conditions, which are described by the Sommerfeld radiation conditions (SRC) at infinity [16] for harmonic medium with flow [15] and for time-space medium without flow [17,18]. While, the exterior boundaries of finite element meshes contains reflection problems [19]. Thus, several techniques have been developed with this purpose such as the Dirichlet-to-Neumann (DtN) operators which based on truncating Fourier expansions, absorbing boundary conditions have been widely studied in the literature [20,21]. While an alternative approach to dealing with truncation of unbounded domains that is represented by Perfectly Matched Layer (PML) method, which was introduced by [22,23]. The reflection problem provided by the PML can be dealt with the use of absorbing layer surrounding the domain of interest. The layer absorbs the scattered field radiated from the outside and does not produce spurious reflections inside of domain [24]. Thirdly, the boundary integral formulations for interior medium with flow require only the discretization of surfaces and generators of three-dimensional and two-dimensional media, respectively. In the other hand, the BEM is applied to determine the acoustic field at any point in the interior region, while the acoustic field by the discretized finite element method is calculated at mesh point.

Additionally, the technique of the Boundary Element Method is directly derived from Helmholtz integral formulation for three-dimensional, two-dimensional and axisymmetric mediums in a mean flow, and it has been employed in several studies [25–27]. The BEM formulation can be developed in a transformed acoustic medium based on the Prandtl–Glauert transformation [28,29] or in an original acoustic domain with and without flow [30,31].

However, the main difficulties of using conventional boundary integral formulations for physical acoustic mediums with a mean flow are the singular problems, the convection effects, and that the radiation conditions. Firstly, it is well known that the boundary integral equation contains singular integrals when a source point is taken at a boundary point of acoustic domain. These singular integrals due to the convected Green's function and its derivatives such as the normal and the flow direction derivatives at singularity [25–27]. Thus, several methods have been proposed to overcome this difficulty. Among them, the Cauchy's principal value (CPV) [32]. For three-dimensional acoustic problems, [33–36] used the polar coordinates system to isolate the singular parts which can be converted into regular parts. An element subdivision method can be used for solving the threedimensional singular integrals which are represented by weakly and strongly singular integrals, and hypersingular kernels [72]. A general algorithm for the direct evaluation of all singular integrals arising in the BEM which based on the standard Gauss quadrature has been proposed in [37,38]. An application of this method has been used in [39] which is based on an even number of Gaussian quadrature points. Moreover, the singular integrations for axisymmetric problems can be evaluated by an analytical method based on the recursive formulations [25,40] and by fast Fourier transform (FFT) techniques [31]. In the Cauchy and Hadamard sense [73], only weakly and strongly singular kernels appearing in the two-dimensional integrals which could be handled by very accurate methods based on analytical integrations [65–67] and transformed numerical integrations [79,80]. Furthermore, [68–71] have used a regular method based on expansion of the singular integrands in polar coordinates for particular cases. Secondly, the convection effects in the presence of a uniform constant flow are due to the convective terms of the convected Green's function and its derivatives. Generally, the convected terms are the normal and the flow direction derivatives for three-dimensional mediums [30], two-dimensional mediums [26,41–43] and axisymmetric mediums [27]. These convective operators lead to a complex form of the Boundary Element Method is strongly dependent on the supplementary terms. Thus, the classical Boundary Element Method formulation with mean flow has an important CPU time compared to the computation time of BEM without flow. In the transformed acoustic medium, the disadvantage of the Boundary Element Method in a Prandtl–Glauert system is the transformation of boundary conditions with mean flow towards boundary conditions without mean flow. The transformed method leads to a complex form of the boundary conditions, which can be handled hardly. Although the main problems posed by the integral formulations in a transformed medium are the same as in the non-flow case [25]. Thirdly, the radiation conditions with mean flow are required for study of the boundary integral formulation in physical unbounded mediums. In the presence of a mean flow, the radiation conditions at infinity have been developed in [16,17,26,28] where the conventional formulations do not explicitly take into account the flow quantities for original acoustic problems. Also, the nonreflecting boundary conditions have been given by [28,77,83] for transformed frequency radiation mediums not satisfying the original integral formulations with mean flow, because these conditions contain transformed variables. The conventional Sommerfeld conditions make difficult to use the BIE formulation for any convected Helmholtz problem.

The aim of this paper is to contribute to reduce the complexity of general BEM formulations for physical time and frequency domains with arbitrary mean flows. Thus, we develop a new improved two-dimensional BEM formulation derived by convolution from an advanced form of a time domain BIE formulation. These formulations are taking into account a new Sommerfeld radiation condition at infinity for acoustic radiation problems with arbitrary mean flows. The phenomenon due to the waves are radiating from finite distance towards infinity to create the total non-reflection of acoustic waves. Through the SRC equation, the proposed BIE formulations are expressed only in terms of the convected Green's kernel and its convected normal derivative. The convected term reduces the flow effects compared to existing BEM formulations. The singular integrals of kernels involved in the integral representation with flow have similar forms of singular terms incorporated in the BEM formulation without flow. They are evaluated analytically in terms of convected angle and numerically in terms of standard Gauss quadrature. For numerical implementation, the CPU time of the improved BEM approach with flow becomes the CPU time of the BEM approach without flow. Moreover, the present formulations significantly reduce the computational burden of the classical formulations such as the conventional BE and FE methods with mean flow.

The outline of this paper is organized as follows. In Section 2 we consider the acoustic radiation and propagation problems in an arbitrary mean flow with these convected wave equations. Thus, we show a new time-space Sommerfeld radiation condition of the total non-reflection acoustic waves at infinity. The objective of Section 3 is to present a methodology to obtain an advanced form of the time domain two-dimensional BEM formulation to solve general acoustic problems. It is necessary to express the two-dimensional Gradient operator for time-space acoustic field in terms of its normal and tangential derivatives. An integral formula derived from the time domain two-dimensional BEM for frequency acoustic domains and when a point source is taken at a boundary point has been proposed. Also, an improved time-harmonic SRC equation with arbitrary mean flow similar to the conventional SRC without flow can be deduced. A numerical implementation of the improved two-dimensional convected boundary integral equation and a new convergence criterion are presented in Section 4. In Section 5, we report the numerical results obtained with our BEM technique that is applied to some realistic wave propagation and radiation problems. The results are validated by comparing the proposed BEM formulation with analytical, conventional and FEM solutions of acoustic sources and an infinite two-dimensional duct using PDtN operators in which these CPU times have been presented.