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Soft coverings and their parameter reductions

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ABSTRACT

Soft set theory is a new mathematical tool to deal with uncertain information. This paper studies soft coverings and their parameter reductions. Firstly, we define a soft set on the power set of an initial universe and discuss its properties. Secondly, we introduce soft coverings and obtain the lattice structure of soft sets induced by them. Thirdly, we investigate parameter reductions of a soft covering by means of attribute reductions in a covering information system and present their algorithm. Finally, we give an application to show the usefulness of parameter reductions of a soft covering.

1. Introduction

There exist various types of uncertainties, imprecision and vagueness in our real life. Classical mathematics do not always successfully deal with the complicated problems with uncertainties. While probability theory, fuzzy set theory [31], rough set theory [21,22], interval mathematics and other mathematical tools are well-known and useful approaches to describe uncertainties. But all these theories have their own limitations, which is possibly due to the inadequacy of parameterization tools associated with these theories. For example, probability theory as a branch of mathematics studying random phenomena and their statistical laws needs to do a lot of test to verify the law of random phenomena; interval mathematics cannot handle information of continuous smooth change; membership functions in fuzzy set theory and upper and lower approximations based on approximation spaces in rough set theory are difficult to be determined. These theories cannot fully express parameters, that is, a large number of parameters cannot be determined.

In 1999, Molodtsov [14] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. This so-called soft set theory is free from the difficulties effecting the existing mathematics tools. Using soft set theory to describe or set objects with traditional mathematics tools is very different. We can describe approximately the original objects in soft set theory. There is no limiting conditions when objects are described. Researchers can choose parameters and their forms according Researchers' needs. The fact that setting parameters is non-binding greatly simplifies decision-making process and then we can still do effective decisions under the circumstances of less information.

Recently there has been a rapid growth of soft set theory. Maji et al. [15–18] studied soft set theory and used this theory to solve some decision making problems. Aktas et al. introduced a notion of soft group. Kong et al. [10] applied soft set approach in decision making problems. Ali et al. [3] defined some new operations on soft sets. Majumdar et al. [19] studied the problem of similarity measurement between soft sets. Feng et al. [7] introduced soft rough sets. Ge et al. [8] discussed relationships between topological spaces and soft sets. Li et al. [11] investigated relationships among soft sets, soft rough sets and topologies.

Rough set theory initiated by Pawlak [21] is a mathematical tool to deal with vagueness and granularity in information systems. This theory is described by equivalence relations over the universe and handles the approximation of an arbitrary subset of the universe by two definable or observable subsets called lower and upper approximations. But in the actual problems, it is hard to obtain equivalence relations between objects, or there is no equivalence relations between objects. So the rough set model

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based on equivalence relations does not fully meet the actual needs.

Covering rough sets [4,24] are an important extension of rough sets. Compared with rough sets, it often gives a more reasonable description to a subset of the universe. In recent years, covering rough set theory has attracted more and more attentions. The works of Zhu et al. [33–36] are fundamental and significant. Specifically, they proposed the concept of the reducible element [35] to solve several problems in covering based rough sets. And this concept has been adopted by numerous researchers [12,25,29,30].

Because rough set theory has its limitations and shortcomings, and soft sets and rough sets describe the different types of uncertainty and can be combined to form a powerful mathematical tool for dealing with uncertain problems, then studying this mathematical tool is importance and necessary so that two theories play their strengths and make up their shortcomings. This will be an important research direction. Actually, a soft set is a parameterized family of subsets of the universe. Soft sets have not any restrictions on the approximate description of objects, and they might form a covering of the universe. Covering rough sets can process data organized by a covering of the universe. Both theories can deal with the uncertainties of data. Therefore, we introduce a new concept of soft coverings and define soft covering rough sets, which are viewed as the combination of soft sets and covering rough sets. This study presents a preliminary, but potentially interesting research direction.

It is worthwhile to mention that parameter reductions of soft sets is a very important problem in soft set theory. The parameter reduction of soft sets means deleting parameters of soft sets which are no or less influence for obtaining the optimal decision, and reducing number of the parameters in decision-makings. Much effort has made on this problem. Maji et al. [16,18] proposed the concept of parameter reduction of soft sets. Chen et al. [5] pointed out that this concept in [18] is unreasonable, and then presented another concept of parameter reduction of soft sets. To overcome the problem of suboptimal choice in [5], Kong et al. [10] introduced the concept of normal parameter reduction of soft sets. But the normal parameter reduction is very complex and the algorithm is hard to understand. Ma et al. [20] investigated the normal parameter reduction and improved this algorithm in [10]. Ali et al. [2] gave another view on parameter reduction of soft sets.

The organization of this paper is as follows: Section 2 briefly reviews some basic concepts about rough sets, soft sets, covering rough sets and lattices; Section 3 defines a soft set on the power set of initial universe and studies some relative properties; Section 4 introduces the concept of soft coverings and presents the upper and lower approximation operators based on a covering approximation space; Section 5 investigates parameter reductions of a soft covering and presents their algorithm; Section 6 gives an application to show the effectiveness and feasibility of parameter reductions of a soft covering; Section 7 summarizes the main points in this paper.

2. Preliminaries

In this section, we briefly recall some basic concepts about rough sets, soft sets, covering rough sets and lattices.

Throughout this paper, U denotes an initial universe, E denotes a set of all possible parameters, 2^U denotes the power set of U and 2^{2^U} denotes the family of all elements of 2^U. We only consider the case where both U and E are nonempty finite sets.

In this paper, "$\bigvee$"(disjunction), "$\bigwedge$" (conjunction), "$\Rightarrow$" (implication), "$\Leftrightarrow$" (biimplication) are propositional connectives in mathematical logic. They are read as "or", "and", "if-then", "if and only if", respectively.

2.1. Rough sets

Let R be an equivalence relation on U. Then a pair (U, R) is called a Pawlak approximation space. Based on (U, R), we can define the following two rough approximations:

$$\underline{R}(X) = \{x \in U \mid [x]_R \subseteq X\},$$

$$\bar{R}(X) = \{x \in U \mid [x]_R \cap X \neq \emptyset\}.$$

$\underline{R}(X)$ and $\bar{R}(X)$ are called the Pawlak lower approximation and the Pawlak upper approximation of X, respectively. In general, we refer to $\bar{R}(X)$ and $\underline{R}(X)$ as Pawlak rough approximations of X.

The Pawlak boundary region of X, denoted by $Bnd_R(X)$, is defined as the difference between Pawlak rough approximations of X, that is, $Bnd_R(X) = \bar{R}(X) - \underline{R}(X)$. It is easy to see that $\underline{R}(X) \subseteq X \subseteq \bar{R}(X)$.

A set is Pawlak rough if its boundary region is not empty, that is, X is Pawlak rough if $\underline{R}(X) \neq \bar{R}(X)$. Otherwise, the set is definable.

We can relax equivalence relations so that rough set theory can be applied to solve more complicated problems in practice. The classical rough set theory based on equivalence relations has been extended to coverings [28,32].

2.2. Covering rough sets

Definition 2.1. Let $\mathcal{C} \subseteq 2^U$.

(1) $\mathcal{C}$ is called a covering of U, if $\bigcup_{K\in\mathcal{C}} K = U$. Furthermore, a covering $\mathcal{C}$ of U is called a partition of U, if $\emptyset \notin C$ and $K \bigcap K' = \emptyset$ for all $K, K' \in \mathcal{C}$.

(2) $(U, \mathcal{C})$ is called a covering approximation space, if $\mathcal{C}$ is a covering of U.

Definition 2.2. [[24]] Let $(U, \mathcal{C})$ be a covering approximation space. For each $X \in 2^U$, put

$$\underline{\mathcal{C}}(X) = \cup\{K | K \in \mathcal{C} \ \ and \ \ K \subseteq X\},$$

$$\bar{\mathcal{C}}(X) = \cup\{K | K \in \mathcal{C} \ \ and \ \ K \cap X \neq \emptyset\}.$$

Then $\underline{\mathcal{C}}(X)$ and $\bar{\mathcal{C}}(X)$ are called the lower approximation and the upper approximation of X with respect to $(U, \mathcal{C})$ or $\mathcal{C}$, respectively. In general, we refer to $\underline{\mathcal{C}}(X)$ and $\bar{\mathcal{C}}(X)$ as rough approximations of X with respect to $(U, \mathcal{C})$ or $\mathcal{C}$.

X is called a definable set if $\underline{\mathcal{C}}(X) = \bar{\mathcal{C}}(X)$. Otherwise, X is called a rough set.

It is easy to show that

$$\underline{\mathcal{C}}(X) = \{x \in U \mid \exists K \in \mathcal{C} \ \ s.t. \ \ x \in K \ \ and \ \ K \subseteq X\},$$

$$\bar{\mathcal{C}}(X) = \{x \in U \mid \exists K \in \mathcal{C} \ \ s.t. \ \ x \in K \ \ and \ \ K \cap X \neq \emptyset\}.$$

2.3. Attribute reductions of coverings

Let $\mathcal{C}$ be a covering of U. For every $x \in U$, denote

$$C(x) = \cap\{C \in \mathcal{C} \mid x \in C\}, \quad Cov(\mathcal{C}) = \{C(x) | \ x \in U\}.$$

Let $\Delta = \{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_n\}$ be a family of coverings of U. For every $x \in U$, denote

$$C_i(x) = \cap\{C \in \mathcal{C}_i \mid x \in C\} \ \ (1 \leq i \leq n),$$

$$\Delta(x) = \cap\{C_i(x) | \ \ 1 \leq i \leq n\}, \qquad Cov(\Delta) = \{\Delta(x) | \ x \in U\}.$$

Definition 2.3. [[27]] Let $\mathcal{C}$ be a covering of U. Then $Cov(\mathcal{C})$ is also a covering of U, we call it the covering of U induced by $\mathcal{C}$.