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Parameterized algorithms for min-max multiway cut and list digraph homomorphism ☆

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ABSTRACT

We design FPT-algorithms for the following problems. The first is List Digraph Homomorphism: given two digraphs G and H, with order n and h, respectively, and a list of allowed vertices of H for every vertex of G, does there exist a homomorphism from G to H respecting the list constraints? Let ℓ be the number of edges of G mapped to non-loop edges of H. The second problem is Min-Max Multiway Cut: given a graph G, an integer $\ell \geq 0$, and a set T of r terminals, can we partition $V(G)$ into r parts such that each part contains one terminal and there are at most ℓ edges with only one endpoint in this part? We solve both problems in time $2^{O(\ell \cdot \log h + \ell^2 \cdot \log \ell)} \cdot n^4 \cdot \log n$ and $2^{O((\ell r)^2 \log \ell r)} \cdot n^4 \cdot \log n$, respectively, via a reduction to a new problem called List Allocation, which we solve adapting the *randomized contractions* technique of Chitnis et al. (2012) [4].

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1. Introduction

The Multiway Cut problem asks, given a graph G, a set of r terminals T, and a non-negative integer ℓ, whether it is possible to partition $V(G)$ into r parts such that each part contains exactly one of the terminals of T and there are at most ℓ edges in total between different parts (i.e., at most ℓ *crossing* edges). In the special case where $|T| = 2$, this gives the classical Minimum Cut problem, which is polynomially solvable [34]. In general, when the number of terminals can be arbitrarily large, the Multiway Cut problem is NP-complete already for $|T| = 3$ [9] (with no restriction on ℓ) and a lot of research has been devoted to the study of this problem and its generalizations, including several classic results on its polynomial approximability [3,15,19,20,26,33].

More recently, special attention to the Multiway Cut problem was given from the parameterized complexity point of view. The existence of an FPT-algorithm for Multiway Cut (when parameterized by ℓ), i.e., an $f(\ell) \cdot n^{O(1)}$-step algorithm, had been a long-standing open problem. This question was answered positively by Marx in [29] with the use of the *important separators* technique, which was also used for the design of FPT-algorithms for several other problems such as

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Directed Multiway Cut [6], Vertex Multicut and Edge Multicut [31], or Vertex Multiway Cut [8]. This technique has been extended to the powerful framework of *randomized contractions* technique, introduced in [4]. This made it possible to design FPT-algorithms for several other problems such as Unique Label Cover, Steiner Cut, or Edge/Vertex Multiway Cut-Uncut. We stress that this technique is quite versatile.

In this paper we use it in order to design FPT-algorithms for parameterizations of two problems that do not seem to be directly related to each other: the Min-Max-Multiway Cut problem [35] and the List Digraph Homomorphism problem.

1.1. Min-Max-Multiway Cut

In the Multiway Cut problem the parameter ℓ bounds the total number of crossing edges (i.e., edges with endpoints in different parts). Svitkina and Tardos [35] considered a "min-max" variant of this problem, namely the Min-Max-Multiway Cut, where ℓ bounds the maximum number of outgoing edges of the parts.[2] In [35], it was proved that Min-Max-Multiway Cut is NP-complete even when the number of terminals is $r = 4$. As a consequence of the results in [35] and [32], Min-Max-Multiway Cut admits a $O(\log^2 n)$-approximation algorithm. This was improved recently in [1] to a $O((\log n \cdot \log r)^{1/2})$-approximation algorithm.

To our knowledge, nothing is known about the parameterized complexity of this problem. We prove the following.

Theorem 1. *There exists an algorithm that solves the* Min-Max-Multiway Cut *problem in* $2^{O((r\ell)^2 \log r\ell)} \cdot n^4 \cdot \log n$ *steps, i.e.,* Min-Max-Multiway Cut *belongs to* FPT *when parameterized by both r and* ℓ.

Throughout the paper, we use $n = |V(G)|$ when we refer to the number of vertices of the graph G in the instance of the considered problem.

1.2. List Digraph Homomorphism

Given two directed graphs G and H, an H-*homomorphism* of G is a mapping $\chi : V(G) \to V(H)$ such that if (x, y) is an arc of G, then $(\chi(x), \chi(y))$ is also an arc in H. In the List Digraph Homomorphism problem, we are given two graphs G and H and a list function $\lambda : V(G) \to 2^{V(H)}$, and we ask whether G has an H-homomorphism such that for every vertex v of G, $\chi(v) \in \lambda(v)$. Graph and digraph homomorphisms have been extensively studied both from the combinatorial and the algorithmic point of view (see e.g., [2,14,16,17,23]).

Specially for the List Digraph Homomorphism problem, a dichotomy characterizing the instantiations of H for which the problem is hard was given in [24] (see also [13]). Notice that the standard parameterization of List Digraph Homomorphism by the size of the graph H is para-NP-complete, as it yields the 3-Coloring problem when G is restricted to be a simple graph and $H = K_3$. A more promising parameterization of List Homomorphism (for undirected graphs) has been introduced in [11], where the parameter is a bound on the number of pre-images of some prescribed set of vertices of H (see also [10,12,30]). Another parameterization, again for the undirected case, was introduced in [5], where the parameter is the number of vertices to be removed from the graph G so that the remaining graph has a list H-homomorphism.

We introduce a new parameterization of List Digraph Homomorphism where the parameter is, apart from $h = |V(H)|$, the number of "crossing edges", i.e., the edges of G whose endpoints are mapped to different vertices of H. For this, we enhance the input with an integer ℓ and ask for a list digraph homomorphism with at most ℓ crossing edges. Clearly, when $\ell = |E(G)|$, this yields the original problem. We call the new problem Bounded List Digraph Homomorphism (in short, BLDH). Notice that the fact that List Digraph Homomorphism is NP-complete even when $h = 3$, implies that BLDH is para-NP-complete when parameterized only by h. The input of BLDH is a quadruple (G, H, λ, ℓ) where G is the guest graph, H is the host graph, $\lambda : V(G) \to 2^{V(H)}$ is the list function, and ℓ is a non-negative integer. Our next step is to observe that BLDH is W[1]-hard, when parameterized only by ℓ. To see this, consider an input (G, k) of the Clique problem and construct the input $(K, \bar{G}, \lambda, \ell)$ where K is the complete digraph on k vertices, $\bar{G}$ is the digraph obtained by G by replacing each edge by two opposite directed arcs between the same endpoints, $\lambda = \{(v, V(G)) \mid v \in V(K)\}$, and $\ell = k(k-1)$. Notice that (G, k) is a yes-instance of Clique if and only if $(K, \bar{G}, \lambda, \ell)$ is a yes-instance of BLDH.

We conclude that when BLDH is parameterized by ℓ or h only, then one may not expect it to be fixed-parameter tractable. This means that the parameterization of BLDH by h and ℓ is meaningful to consider. Our result is the following.

Theorem 2. *There exists an algorithm that solves the* Bounded List Digraph Homomorphism *problem in* $2^{O(\ell \cdot \log h + \ell^2 \cdot \log \ell)} \cdot n^4 \cdot \log n$ *steps, i.e.,* Bounded List Digraph Homomorphism *belongs to* FPT *when parameterized by the number* ℓ *of crossing edges and the number h of vertices of H.*

[2] Notice that under this viewpoint Multiway Cut can be seen as Min-Sum-Multiway Cut.

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