Journal of Computer and System Sciences 85 (2017) 168–182

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

Editing to a planar graph of given degrees ☆

Konrad K. Dabrowski [a,*], Petr A. Golovach [b], Pim van 't Hof [c], Daniël Paulusma [a], Dimitrios M. Thilikos [d,e,f]

[a] School of Engineering and Computing Sciences, Durham University, United Kingdom
[b] Department of Informatics, University of Bergen, Norway
[c] School of Built Environment, Rotterdam University of Applied Sciences, Rotterdam, The Netherlands
[d] Computer Technology Institute and Press "Diophantus", Patras, Greece
[e] Department of Mathematics, National and Kapodistrian University of Athens, Athens, Greece
[f] AlGCo project-team, CNRS, LIRMM, Montpellier, France

ARTICLE INFO

Article history:
Received 21 February 2016
Received in revised form 17 November 2016
Accepted 26 November 2016
Available online 1 December 2016

Keywords:
Graph editing
Connected graph
Planar graph
Polynomial kernel

ABSTRACT

We consider the following graph modification problem. Let the input consist of a graph $G = (V, E)$, a weight function $w: V \cup E \to \mathbb{N}$, a cost function $c: V \cup E \to \mathbb{N}_0$ and a degree function $\delta: V \to \mathbb{N}_0$, together with three integers k_v, k_e and C. The question is whether we can delete a set of vertices of total weight at most k_v and a set of edges of total weight at most k_e so that the total cost of the deleted elements is at most C and every non-deleted vertex v has degree $\delta(v)$ in the resulting graph G'. We also consider the variant in which G' must be connected. Both problems are known to be NP-complete and W[1]-hard when parameterized by $k_v + k_e$. We prove that, when restricted to planar graphs, they stay NP-complete but have polynomial kernels when parameterized by $k_v + k_e$.

1. Introduction

Graph modification problems capture a variety of fundamental graph-theoretic problems, and as such they are very well studied in algorithmic graph theory. The aim is to modify some given graph G into some other graph H, that satisfies a *certain property*, by applying at most some given number operations from a set S of *prespecified graph operations*. Well-known graph operations are the edge addition, edge deletion and vertex deletion, denoted by ea, ed and vd, respectively. For example, if $S = \{\text{vd}\}$ and H must be a clique or independent set, then we obtain two basic graph problems, namely CLIQUE and INDEPENDENT SET, respectively. To give a few more examples, if H must be a forest and either $S = \{\text{ed}\}$ or $S = \{\text{vd}\}$, then we obtain the problems FEEDBACK EDGE SET and FEEDBACK VERTEX SET, respectively. As we discuss in detail later, it is also common to consider sets S consisting of more than one graph operation.

☆ An extended abstract of this paper appeared in the proceedings of CSR 2015 [10]. The first and fourth authors were supported by EPSRC Grant EP/K025090/1. The research of the second author received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007–2013)/ERC Grant Agreement n. 267959. The research of the fifth author was co-financed by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) – Research Funding Program: ARISTEIA II.

* Corresponding author.
E-mail addresses: konrad.dabrowski@durham.ac.uk (K.K. Dabrowski), petr.golovach@ii.uib.no (P.A. Golovach), p.van.t.hof@hr.nl (P. van 't Hof), daniel.paulusma@durham.ac.uk (D. Paulusma), sedthilk@thilikos.info (D.M. Thilikos).

http://dx.doi.org/10.1016/j.jcss.2016.11.009

A property is *hereditary* if it is closed under taking induced subgraphs. A property is *non-trivial* if it is both true for infinitely many graphs and false for infinitely many graphs. A classic result of Lewis and Yannakakis [24] is that the vertex deletion problem is NP-hard for any property that is both hereditary and non-trivial. In an earlier paper, Yannakakis [33] also showed NP-hardness results for the edge deletion problem for several properties, such as being planar or outer-planar. Natanzon, Shamir and Sharan [29] and Burzyn, Bonomo and Durán [5] proved that the graph modification problem is NP-complete for several target hereditary graph properties when $S = \{\mathsf{ea}, \mathsf{ed}\}$.

As we can see from the above results, graph modification problems are often intractable even for elementary cases when $S \subseteq \{\mathsf{ea}, \mathsf{ed}\}$. As such, many papers in this area study the complexity of graph modification problems when parameterized by the total number of permitted operations k.

Cai [6] proved that the graph modification problem is FPT when parameterized by k, if $S = \{\mathsf{ea}, \mathsf{ed}, \mathsf{vd}\}$ and the desired property is that of belonging to any fixed graph class characterized by a finite set of forbidden induced subgraphs. Khot and Raman [21] determined all non-trivial hereditary properties for which the vertex deletion problem is FPT on n-vertex graphs with parameter $n - k$ and proved that the problem is W[1]-hard with respect to this parameter for all other such properties.

From the aforementioned results we see that the graph modification problem has been thoroughly studied for hereditary properties. Several other natural types of properties have also been considered. For instance, Dabrowski et al. [9] combined a number of previous results [4,7,8] with new results to give a complete classification of the (parameterized) complexity of the problem of modifying an input graph into a connected graph where each vertex has some prescribed degree parity for every set $S \subseteq \{\mathsf{ea}, \mathsf{ed}, \mathsf{vd}\}$.

1.1. Our focus

In this paper we consider the case when the vertices of the resulting graph must satisfy some prespecified degree constraints (note that such properties are non-hereditary, so the result of Lewis and Yannakakis does not apply to this case). This is a natural direction to consider given the classical structural results [25,32] on so-called f-factors in graphs, which are spanning subgraphs in which each vertex u must have degree $f(u)$ for some specified function f (these results immediately imply that an f-factor in a graph can be found in polynomial time if one exists, while finding connected f-factors, e.g. Hamilton cycles, is NP-complete).

Before presenting our results, we briefly discuss the known results and the general framework they fall under.

General framework. Moser and Thilikos in [28] and Mathieson and Szeider [27] initiated an investigation into the parameterized complexity of graph modification problems with respect to degree constraints. This leads to the following general problem.

> Degree Constraint Editing(S)
> *Instance:* A graph G, integers d, k and a function $\delta \colon V(G) \to \{1, \dots, d\}$.
> *Question:* Can G be modified into a graph G' such that $d_{G'}(v) = \delta(v)$ for each $v \in V(G')$ using at most k operations from the set S?

Mathieson and Szeider [27] classified the parameterized complexity of this problem for $S \subseteq \{\mathsf{ea}, \mathsf{ed}, \mathsf{vd}\}$. In particular they showed the following results. If $S \subseteq \{\mathsf{ea}, \mathsf{ed}\}$ then the problem is polynomial-time solvable. If $\mathsf{vd} \in S$ then the problem is NP-complete, W[1]-hard with parameter k and FPT with parameter $d + k$. Moreover, they proved that the latter result holds even for a more general version, in which the vertices and edges have costs and the desired degree for each vertex should be in some given subset of $\{1, \dots, d\}$. If $\{v\} \subseteq S \subseteq \{\mathsf{ed}, \mathsf{vd}\}$, they proved that the problem has a polynomial kernel when parameterized by $d + k$ even if vertices and edges have costs. Recently, Mathieson [26] considered graph editing problems for a number of alternative forms of degree constraints. Golovach [19] considered the cases $S = \{\mathsf{ea}, \mathsf{vd}\}$ and $S = \{\mathsf{ea}, \mathsf{ed}, \mathsf{vd}\}$ and proved (amongst other results) that for these cases the problem has no polynomial kernel when parameterized by $d + k$ unless NP $\subseteq$ coNP/poly. Froese, Nichterlein and Niedermeier [14] gave more kernelization results for Degree Constraint Editing(S).

Golovach [18] introduced a variant of Degree Constraint Editing(S) with the extra condition that the resulting graph must be *connected*. He proved that, for $S = \{\mathsf{ea}\}$, this variant is NP-complete, FPT when parameterized by k, and has a polynomial kernel when parameterized by $d + k$. The connected variant is readily seen to be W[1]-hard when $\mathsf{vd} \in S$ by a straightforward modification of the proof of the W[1]-hardness result for Degree Constraint Editing(S), when $\mathsf{vd} \in S$, as given by Mathieson and Szeider [27].

Our results. In the light of the above NP-completeness and W[1]-hardness results when $\mathsf{vd} \in S$ it is natural to restrict the input graph G to a special graph class. Hence, inspired by the above results, we consider the set $S = \{\mathsf{ed}, \mathsf{vd}\}$ and study both variants of these problems (where we insist that the resulting graph G' is connected and where we do not) for *planar* input graphs. The problem variant not demanding connectivity is defined as follows. (In fact the problems we study are slightly more general.)