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Graph modification problem for some classes of graphs

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ABSTRACT

The issue of determining the complexity of modification problems for chordal bipartite graphs has been raised multiple times in the literature. We show that the completion and deletion problems for chordal bipartite graphs are NP-hard. The corresponding problems for weakly chordal graphs are already known to be hard. As a byproduct, we obtain results on modification problems for a variety of related classes of graphs and also simplify the arguments for the class of weakly chordal graphs.

1. Introduction

Given a property Π and a graph G, the Π completion problem asks for the fewest number of edges needed to be added to G so that the resulting graph has property Π. The Π deletion problem is defined analogously but only deletion of edges is allowed. In the Π editing problem, defined similarly, deletion as well as addition of edges is allowed. We consider the decision version of the problems in which, in addition to Π and G, we are given a positive integer k and the problem is to determine whether at most k changes to G of the allowed type are enough so that the resulting graph has property Π. We use $\ll G, k \gg$ to refer to an instance of the decision version of Π completion, deletion, or editing problem.

We use P_k to refer to an induced path on k vertices and C_k to refer to an induced cycle on k vertices. In graph $G = (V, E)$ for $S \subseteq V$, $G[S]$ refers to the subgraph of G induced by S. A *hole* is a C_k, $k \geq 5$. A *house* is the complement of P_5. A *domino* is the bipartite graph obtained from a cycle on six vertices by adding exactly one chord. We use *graph G contains graph H* to mean H is an induced subgraph of G.

A graph is *chordal* if it does not contain any C_k, $k \geq 4$. For $n \geq 3$, a *sun* is a chordal graph with the Hamiltonian cycle $(x_1 y_1 x_2 y_2 \ldots x_n y_n)$ in which each x_i is of degree exactly two and the set of y_i vertices induces a complete graph. A graph is *strongly chordal* if it is chordal and does not contain any sun [6]. A graph is *split* if its vertex set can be partitioned into a clique and an independent set. A graph is *hh-free* if it does not contain any house or a hole. A graph is *hhd-free* if it does not contain any house, hole, or a domino. A graph is *hole-free* if it does not contain any holes. A graph is *weakly chordal* if neither the graph nor its complement contains any holes. A bipartite graph is *chordal bipartite* if it does not contain any induced cycles on 6 or more vertices; equivalently, a chordal bipartite graph is precisely a bipartite graph that is also weakly chordal.

These classes of graphs are well-studied and the inclusions among these graph classes are well known [2]. For example, every split graph is chordal, every chordal graph is hhd-free, every hhd-free graph is hh-free, and every hh-free graph is hole-free as well as weakly chordal [2]. Further, each of the mentioned classes can be recognized in polynomial time [2].

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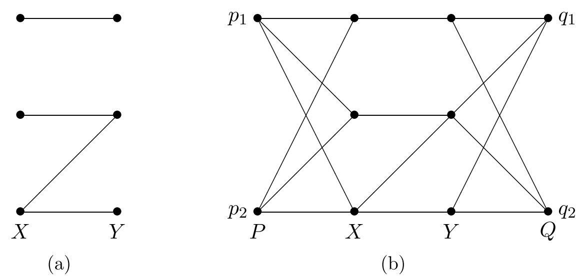

Fig. 1. (a) Graph (X, Y, E) and $k = 1$. (b) Graph $T(X, Y, E, k)$.

Yannakakis defined [13,14] a bipartite graph (X, Y, E) to be a *chain graph* if vertices in X (Y) can be totally ordered based on containment of neighborhoods of vertices; thus, for $v, w \in X$ $(v, w \in Y)$ either $N(v) \subseteq N(w)$ or $N(w) \subseteq N(v)$ holds. Yannakakis showed [14] that a bipartite graph is a chain graph if and only if it does not contain a $2K_2$.

Graph modification problems have been considered for a variety of properties. Yannakakis [13] proved that chordal completion is NP-complete. A tool that was used in [13] is to show that chain completion is NP-complete. The chain deletion problem, defined analogously, was shown to be NP-complete in [11]. For a variety of graph classes, hardness results for graph modification problems were presented in [3].

The issue of determining the complexity of modification problems for chordal bipartite graphs has been raised multiple times in the literature [10,8,9,7]. The complexities of the related sandwich problem for chordal bipartite graphs and minimal completions to chordal bipartite graphs were considered in [5,12,7]. We show that the completion and deletion problems for chordal bipartite graphs are NP-hard. The corresponding problems for weakly chordal graphs are already known to be hard [3]. As a byproduct, we obtain results on modification problems for a variety of related classes of graphs and also simplify the arguments for the class of weakly chordal graphs. Specifically we show that the completion problem is NP-complete for each of the following classes of graphs: chordal bipartite, C_6-free bipartite, hh-free, strongly chordal, and split strongly chordal. We show that the deletion problem is NP-complete for each of the following classes of graphs: chordal bipartite, C_6-free bipartite, hh-free, hole-free, strongly chordal, and split strongly chordal. We also show that the chordal bipartite editing problem is at least as hard as the chain editing problem; the complexity of the latter problem is currently open.

We assume, as in the definition of the chain completion (deletion) problem, that in an instance $\ll (X, Y, E), k \gg$ of the chordal bipartite completion (deletion) problem we are given a bipartition X, Y of the vertex-set of the graph. Note that the minimum number of edges needed to be added to make a bipartite graph chordal bipartite is invariant under different bipartitions; thus, our assumption is valid in the context of the modification problems for chordal bipartite. Clearly, the specific bipartition given does not matter for the case of the deletion problem.

1.1. A construction

We describe a construction that is used in a majority of our arguments.

Given bipartite graph $G = (X, Y, E)$ with the bipartition X, Y, and positive integer k, we define the graph $T(X, Y, E, k) = (V' = P \cup X \cup Y \cup Q, E')$ where:

$P = \{p_1, p_2, \cdots, p_{k+1}\}$,
$Q = \{q_1, q_2, \cdots, q_{k+1}\}$, and
$E' = E \cup \{px \mid p \in P \text{ and } x \in X\} \cup \{qy \mid q \in Q \text{ and } y \in Y\}$.

Note that $T(X, Y, E, k)$ is bipartite with $X \cup Q$, $Y \cup P$ as a bipartition. The construction is illustrated in Fig. 1.

When we refer to $T(X, Y, E, k)$ as a graph we use the notation $(V' = P \cup X \cup Y \cup Q, E')$ and when we refer to it as a bipartite graph with a specified bipartition we use the notation $(X \cup Q, Y \cup P, E')$.

2. Deletion problems

Lemma 2.1. *Chordal bipartite deletion is NP-complete.*

Proof. As chordal bipartite graphs can be recognized in polynomial time, the problem is in NP.

Given instance $\ll (X, Y, E), k \gg$ of chain deletion, construct instance $\ll T(X, Y, E, k) = (V' = P \cup X \cup Y \cup Q, E'), k \gg$ of chordal bipartite deletion.

Suppose $G = (X, Y, E - F)$ with $|F| \leq k$ is a chain graph. We claim that $H = (V', E' - F)$ is chordal bipartite. Clearly H is bipartite. Now suppose C is a chordless cycle in H with $|C| \geq 6$. Observe that for any two vertices x, y on C we have $N(x) \nsubseteq N(y)$ and $N(y) \nsubseteq N(x)$. On the other hand, in H, for $x, y \in P$ (or in X, or in Y, or in Q), we have $N(x) \subseteq N(y)$ or $N(y) \subseteq N(x)$. Thus, C has at most one vertex from each of X, Y, P, and Q and $|C| \leq 4$, a contradiction.

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