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Solving the canonical representation and Star System Problems for proper circular-arc graphs in logspace ☆

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ABSTRACT

We present a logspace algorithm that constructs a canonical intersection model for a given proper circular-arc graph, where *canonical* means that isomorphic graphs receive identical models. This implies that the recognition and the isomorphism problems for these graphs are solvable in logspace. For the broader class of concave-round graphs, which still possess (not necessarily proper) circular-arc models, we show that a canonical circular-arc model can also be constructed in logspace. As a building block for these results, we design a logspace algorithm for computing canonical circular-arc models of circular-arc hypergraphs. This class of hypergraphs corresponds to matrices with the *circular ones property*, which play an important role in computational genomics. Our results imply that there is a logspace algorithm that decides whether a given matrix has this property.

Furthermore, we consider the Star System Problem that consists in reconstructing a graph from its closed neighborhood hypergraph. We show that this problem is solvable in logarithmic space for the classes of proper circular-arc, concave-round, and co-convex graphs.

Note that solving a problem in logspace implies that it is solvable by a parallel algorithm of the class AC^1. For the problems under consideration, at most AC^2 algorithms were known earlier.

1. Introduction

With a family of sets $\mathcal{H}$ we associate the *intersection graph* $\mathbb{I}(\mathcal{H})$ on vertex set $\mathcal{H}$ where two sets $A, B \in \mathcal{H}$ are adjacent if and only if they have a nonempty intersection. We call $\mathcal{H}$ an *intersection model* of a graph G if G is isomorphic to $\mathbb{I}(\mathcal{H})$. Any isomorphism from G to $\mathbb{I}(\mathcal{H})$ is called a *representation* of G by an intersection model. If $\mathcal{H}$ consists of intervals (resp. arcs of a circle), it is also referred to as an *interval model* (resp. an *arc model*). An intersection model $\mathcal{H}$ is *proper* if the sets in $\mathcal{H}$ are pairwise incomparable by inclusion. G is called a *(proper) interval graph* if there is a (proper) interval model of G. The classes of *circular-arc* and *proper circular-arc* graphs are defined similarly. Throughout the paper we will use the shorthands *CA* and *PCA*, respectively.

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We design a logspace algorithm that for a given PCA graph computes a canonical representation by a proper arc model, where *canonical* means that isomorphic graphs receive identical models. Note that this algorithm provides a simultaneous solution in logspace of both the recognition and the isomorphism problems for the class of PCA graphs.

In [21], along with Bastian Laubner we gave a logspace solution for the canonical representation problem of proper interval graphs. Though PCA graphs may at first glance appear close relatives of proper interval graphs, the extension of the result of [21] achieved here is far from being straightforward. Combinatorial differences between these two classes of graphs are well known, and they are responsible for the fact that algorithms for PCA graphs often need new ideas and are much more involved than the algorithms for the same problems on proper interval graphs; cf. [8,10,11,15,20,27,28,30,35]. One combinatorial difference, very important in our context, lies in the relationship of these graph classes to interval and circular-arc hypergraphs that we will explain shortly.

An *interval hypergraph* is a hypergraph isomorphic to a system of intervals of integers. A *circular-arc (CA) hypergraph* is defined similarly if, instead of integer intervals, we consider arcs in a discrete circle. With any graph G, we associate its *closed neighborhood hypergraph* $\mathcal{N}[G] = \{N[v]\}_{v\in V(G)}$ on the vertex set of G, where for each vertex v we have the hyperedge $N[v]$ consisting of v and all the vertices adjacent to v. Roberts [33] discovered that G is a proper interval graph if and only if $\mathcal{N}[G]$ is an interval hypergraph. The circular-arc world is more complex. While $\mathcal{N}[G]$ is a CA hypergraph whenever G is a PCA graph, the converse is not always true. PCA graphs are properly contained in the class of those graphs whose neighborhood hypergraphs are CA. Graphs with this property are called *concave-round* by Bang-Jensen, Huang, and Yeo [3] and *Tucker graphs* by Chen [7]. The latter name is justified by Tucker's result [38] saying that all these graphs are CA (although not necessarily proper CA). Hence, it is natural to consider the problem of constructing arc representations for concave-round graphs. We solve this problem in logspace and also in a canonical way.

Our working tool is a logspace algorithm for computing canonical representations of CA hypergraphs. This algorithm can also be used to test in logspace whether a given Boolean matrix has the *circular ones property*, that is, whether the columns can be permuted so that the 1-entries in each row form a segment up to a cyclic shift. Note that a matrix has this property if and only if it is the incidence matrix of a CA hypergraph. The recognition problem of the circular ones property arises in computational biology, namely in analysis of circular genomes [14,31].

Our techniques are also applicable to the *Star System Problem* where, for a given hypergraph $\mathcal{H}$, we have to find a graph G such that $\mathcal{H} = \mathcal{N}[G]$, if such a graph exists. In the restriction of the problem to a class of graphs C, we seek for G only in C. We give logspace algorithms solving the Star System Problem for PCA and for concave-round graphs.

1.1. Comparison with previous work

Recognition, model construction, and isomorphism testing. The recognition problem for PCA graphs, along with model construction, was solved in linear time by Deng, Hell, and Huang [11], by Kaplan and Nussbaum [20], and by Soulignac [36]; and in AC^2 by Chen [8]. Note that linear-time and logspace results are in general incomparable, while the existence of a logspace algorithm for a problem implies that it is solvable in AC^1. The isomorphism problem for PCA graphs was solved in linear time by Lin, Soulignac, and Szwarcfiter [27]; their algorithm computes canonical representations. Curtis et al. give a linear time isomorphism test for the larger class of concave-round graphs [10].

Chen [7] showed that the isomorphism problem for concave-round graphs is in AC^2. Circular-arc models for concave-round graphs were known to be constructible also in AC^2 (Chen [6]).

Extending these upper bounds to the class of all CA graphs remains a challenging problem. While this class can be recognized in linear time by McConnell's algorithm [30] (along with constructing an intersection model), no polynomial-time isomorphism test for CA graphs is currently known (see the discussion in [10], where a counterexample to the correctness of Hsu's algorithm [16] is given). This provides further evidence that CA graphs are algorithmically harder than interval graphs. For the latter class we have linear-time algorithms for recognition [4] and canonical representation [29] due to the seminal work by Booth and Lueker; logspace algorithms for these tasks are designed in [21].

The aforementioned circular ones property and the related *consecutive ones property* (requiring that the columns can be permuted so that the 1-entries in each row form a segment) were studied in [4,17,18], where linear-time algorithms are given; parallel AC^2 algorithms were suggested in [9,2].

Star System Problem. The decision version of the Star System Problem for general graphs is NP-complete (Lalonde [25]). It stays NP-complete if restricted to non-co-bipartite graphs (Aigner and Triesch [1]) or to H-free graphs for H being a cycle or a path on at least 5 vertices (Fomin et al. [13]). The restriction to co-bipartite graphs has the same complexity as the general graph isomorphism problem [1]. Polynomial-time algorithms are known for H-free graphs for H being a cycle or a path on at most 4 vertices [13] and for bipartite graphs (Boros et al. [5]). An analysis of the algorithms in [13] for C_3- and C_4-free graphs shows that the Star System Problem for these classes is solvable even in logspace, and the same holds true for the class of bipartite graphs; see [22]. Moreover, the problem is solvable in logspace for any logspace-recognizable class of C_4-free graphs, in particular, for chordal, interval, and proper interval graphs; see [22].

2. Basic definitions

We use the standard graph-theoretic terminology as, e.g., in [12]. The vertex set of a graph G is denoted by $V(G)$. The *complement of a graph G* is the graph $\overline{G}$ with $V(\overline{G}) = V(G)$ such that two vertices are adjacent in $\overline{G}$ if and only if they are

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