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A novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence☆

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ABSTRACT

This paper proposes a novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. First, the uncertain degrees of various parameters are determined via grey relational analysis, which is applied to calculate the grey mean relational degree. Second, suitable mass functions of different independent alternatives with different parameters are given according to the uncertain degree. Third, to aggregate the alternatives into a collective alternative, Dempster's rule of evidence combination is applied. Finally, the alternatives are ranked and the best alternatives are obtained. The effectiveness and feasibility of this approach are demonstrated by comparing with the mean potentiality approach because the measure of performance of this approach is the same as the mean potentiality approach's, the belief measure of the whole uncertainty falls from 0.4723 to 0.0782 (resp. 0.3821 to 0.0069) in the example of Section 5 (resp. Section 6).

1. Introduction

To solve complicated problems in economics, engineering, environmental science and social science, classical mathematical methods are not always successful because of various types of uncertainties present in these problems. There are several theories: probability theory, fuzzy set theory [30], rough set theory [22] and interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. To overcome these kinds of difficulties, Molodtsov [19] proposed soft set theory for modeling uncertainty.

Recently works on soft set theory are progressing rapidly. Maji et al. [20] defined fuzzy soft sets by combining soft sets with fuzzy sets, in other words, a degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. The study of hybrid models combining soft sets or fuzzy soft sets with other mathematical structures and new operations are emerging as an active research topic of soft set theory [10,29]. Aktas et al. [1] initiated soft groups. Jun applied soft sets to BCK/BCI-algebras [9]. Jiang et al. [10] extended soft sets with description logics. Li et al. [18] investigated relationships among soft sets, soft rough sets and topologies.

At the same time, there have been some progress concerning applications of soft set theory, especially the usage of soft sets in decision making. Using soft set theory to describe or set objects with traditional mathematics tools is very different. We can describe approximately the original objects in soft set theory. There is no limiting condition when objects are described. Researchers can choose parameters and their forms according needs. The fact that setting parameters is non-binding greatly simplifies decision-making process and then we can still do effective decisions under the circumstance of the absence of partial information.

Maji et al. [21] first applied soft sets to solve decision making problems by means of rough set theory. Chen et al. [3] defined the parameterization reduction of soft set and discussed its application of decision making problem. Cağman et al. [4] constructed an uni-int decision making method which selects a set of optimum elements from the alternatives by using uni-int decision functions. Roy et al. [23] discussed score value as the evaluation basis to finding an optimal choice object in fuzzy soft sets. Kong et al. [13] argued that the Roy's method was incorrect in general and they proposed a revised algorithm. Feng et al. [7] applied level soft sets to discuss fuzzy soft sets based decision making. Jiang et al. [11] generalized the adjustable approach to fuzzy soft sets based decision making and presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic

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fuzzy soft sets. Based on Feng' works, Basu et al. [2] further investigated the previous methods to fuzzy soft sets in decision making and introduced the mean potentiality approach, which was showed more efficient and more accurate than the previous methods.

The existing approaches have significant contributions to solve fuzzy soft sets in decision making. However, these approaches are mainly based on the level soft set, and the decision makers select any level soft set with much subjectivity and uncertainty [2]. Moreover, there exists no unique or uniform criterion for the selection, the same decision problem may induce many different results by using different evaluation criteria. As a result, it is difficult to judge that which result is adequate, and which method or level soft sets should be chosen for selecting the optimal choice object. The key to this problem is how to reduce subjectivity and uncertainty when we choose making decisions method. Then it is necessary to pay attention to this issue.

Grey relational analysis initiated by Deng [6] is utilized for generalizing estimates under small samples and uncertain conditions, and it can be regarded an effective method to solve decision making problems [12,26,33]. Dempster–Shafer theory of evidence is a new important reasoning method under uncertainty, which has an advantage to deal with subjective judgments and to synthesize the uncertainty of knowledge [32].

Compared to probability theory, Dempster–Shafer theory of evidence [5,24] can capture more information to support decision-making by identifying the uncertain and unknown evidence. It provides a mechanism to derive solutions from various vague evidences without knowing much prior information and has been successfully applied into many fields such as intelligent medical diagnosis [8], knowledge reduction [27], fault diagnosis [28], multi-class classification [17], supplier selection [25], etc. Moreover, applying both theories enables the ultimate decision makers to take advantage of both methods' merits and make evaluation experts to deal with uncertainty and risk confidently [16,25]. The hybrid model has been proved to have its usefulness and versatility in successfully solving a variety of problems in the information sciences, such as data mining, knowledge discovery, and decision making.

Therefore, it is very meaningful to explore an approach to fuzzy soft set in decision making by combining Dempster–Shafer theory of evidence with grey relational analysis.

The purpose of this paper is to give a novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. The novelty aspects or advantages of this approach is avoiding the selection of appropriate level soft sets and distribution of parameters' weight, reducing significantly the uncertainty of decision-making and the fuzziness of people's subjective understanding, improving the reliability of decision making and increasing the level of decision-making.

2. Preliminaries

Throughout this paper, U denotes an initial universe, E denotes the set of all possible parameters, I^U denotes the family of all fuzzy sets in U. We only consider the case where U and E are both nonempty finite sets.

In this section, we briefly recall some basic concepts about fuzzy soft sets, the measure of performance of methods and Dempster–Shafer theory of evidence.

2.1. Fuzzy soft sets

Definition 2.1 *([20])*. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F: A \to I^U$.

Table 1
Tabular representation of the fuzzy soft set (F, A).

	h_1	h_2	h_3	h_4	h_5	h_6
e_1	0.6	1	0.2	0.3	1	0.7
e_2	1	0.5	0.3	0.2	0.1	0.9
e_3	0.1	0.4	0.8	1	0	0.1
e_4	0.1	0.3	1	0.9	0	1

In other words, a fuzzy soft set (F, A) over U is a parametrized family of fuzzy sets in the universe U.

Example 2.2. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $A = \{e_1, e_2, e_3, e_4\}$. Let (F, A) be a fuzzy soft set over U, defined as follows:

$$F(e_1) = \left\{ \frac{h_1}{0.6}, \frac{h_2}{1}, \frac{h_3}{0.2}, \frac{h_4}{0.3}, \frac{h_5}{1}, \frac{h_6}{0.7} \right\},$$

$$F(e_2) = \left\{ \frac{h_1}{1}, \frac{h_2}{0.5}, \frac{h_3}{0.3}, \frac{h_4}{0.2}, \frac{h_5}{0.1}, \frac{h_6}{0.9} \right\},$$

$$F(e_3) = \left\{ \frac{h_1}{0.1}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{1}, \frac{h_5}{0}, \frac{h_6}{0.1} \right\},$$

$$F(e_4) = \left\{ \frac{h_1}{0.1}, \frac{h_2}{0.3}, \frac{h_3}{1}, \frac{h_4}{0.9}, \frac{h_5}{0}, \frac{h_6}{1} \right\}.$$

Then (F, A) is described by Table 1.

It is easy to see that every soft set may be considered as a fuzzy soft set (see [7]).

Definition 2.3 *([20])*. Let $A, B \subseteq E$. Let $((F, A)$ and (G, B) be two fuzzy soft sets over U. Then "(F, A) AND (G, B)" is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for $\alpha \in A$ and $\beta \in B$.

2.2. Measure of performance of methods

Definition 2.4 *([2])*. The measure of performance of the method (M) which satisfies the optimality criteria to solve a fuzzy soft set in decision making is defined as follows

$$\Upsilon_M = \frac{1}{\sum_{i=1}^{m} \sum_{j=1, i \neq j}^{m} |F(e_i)(O_p) - F(e_j)(O_p)|} + \sum_{i=1}^{m} F(e_i)(O_p),$$

where m is the number of choice parameters and $F(e_i)(O_p)$ is the membership value of the optimal object O_p for the choice parameter e_i.

Suppose there are two methods M_1, M_2 which satisfy the optimality criteria and their measure of performances are respectively Υ_{M_1} and Υ_{M_2}. If $\Upsilon_{M_1} > \Upsilon_{M_2}$, then M_1 is better than M_2. If $\Upsilon_{M_1} < \Upsilon_{M_2}$, then M_2 is better than M_1. If $\Upsilon_{M_1} = \Upsilon_{M_2}$, then the performance of the both methods are the same.

2.3. Dempster–Shafer theory of evidence

Dempster–Shafer theory of evidence is a new important reasoning method under uncertainty. It has an advantage to deal with subjective judgments and to synthesize the uncertainty knowledge (see [32]). This theory discusses a frame of discernment, denoted by Θ, which is a finite nonempty set of mutually exclusive and exhaustive hypotheses (or all possible outcomes of an event), denoted by $\{A_1, A_2, \ldots, A_n\}$. 2^Θ denotes the set of all subsets of Θ.

The independence of evidences does not hold in many cases and people emphasize the independence of evidences in applications of Dempster–Shafer theory of evidence. Thus researchers ideally assume the independence of evidences in using Dempster–Shafer theory of evidence.