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Belief, knowledge, lies and other utterances in an algebra for space and extrusion ☆

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ABSTRACT

The notion of *constraint system (cs)* is central to declarative formalisms from *concurrency theory* such as process calculi for concurrent constraint programming (ccp). Constraint systems are often represented as *lattices*: their elements, called *constraints*, represent partial information and their order corresponds to entailment. Recently a notion of *n-agent spatial cs* was introduced to represent information in concurrent constraint programs for spatially distributed multi-agent systems. From a *computational* point of view a spatial constraint system can be used to specify partial information holding in a given agent's space (*local information*). From an *epistemic* point of view a spatial cs can be used to specify information that a given agent considers true (*beliefs*). Spatial constraint systems, however, do not provide a mechanism for specifying the mobility of information/processes from one space to another. Information mobility is a fundamental aspect of concurrent systems.

In this article we develop the theory of spatial constraint systems with operators to specify information and processes moving from a space to another. We shall investigate the properties of this new family of constraint systems and illustrate their applications. From a computational point of view the new operators provide for process/information *extrusion*, a central concept in formalisms for *mobile communication*. From an epistemic point of view extrusion corresponds to a notion we shall call *utterance*; a piece of information that an agent communicates to others but that may be inconsistent with the agent's beliefs. Utterances can then be used to express instances of epistemic notions such as *hoaxes* or intentional *lies*. Spatial constraint system can express the epistemic notion of *belief* by means of space functions that specify *local information*. We shall show that spatial constraint can also express the epistemic notion of knowledge by means of a derived spatial operator that specifies *global information*.

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1. Introduction

Epistemic, mobile and spatial behavior are common place in today's distributed systems. The intrinsic *epistemic* nature of these systems arises from social behavior. Most people are familiar with digital systems where users share their *beliefs*, *opinions* and even intentional *lies* (hoaxes). Also, systems modeling decision behavior must account for those decisions' dependence on the results of interactions with others within some social context. The courses of action stemming from some agent decision result not only from the rational analysis of a particular situation but also from the agent beliefs or information that sprang from the interactions with other participants involved in that situation. Appropriate performance within these social contexts requires the agent to form beliefs about the beliefs of others. Spatial and mobile behavior is exhibited by apps and data moving across (possibly nested) spaces defined by, for example, friend circles, groups, and shared folders. We therefore believe that a solid understanding of the notion of *space* and *spatial mobility* as well as the flow of epistemic information is relevant in any model of today's distributed systems.

Declarative formalisms of concurrency theory such as process calculi for *concurrent constraint programming* (ccp) [39] were designed to give explicit access to the concept of partial information and, as such, have close ties with logic [33,30]. This makes them ideal for the incorporation of epistemic and spatial concepts by expanding the logical connections to include *multi-agent modal logic* [27]. In fact, the sccp calculus [26] extends ccp with the ability to define local computational spaces where agents can store epistemic information and run processes.

1.1. Problem: spatial and epistemic mobility

Despite being able to express meaningful epistemic and spatial phenomena such as belief, local and global information, the sccp calculus does not provide a mechanism to intentionally *extrude* information or processes from local spaces. Such a mechanism would allow sccp to express the transfer of epistemic information from one space into another.

Constraint Systems. The notion of *constraint system (cs)* is central to ccp and other declarative formalisms such as (concurrent) constraint logic programming (clp). All ccp calculi are parametric in a cs that specifies partial information upon which programs (processes) may act. A cs is often represented as a *complete lattice* $(Con, \sqsubseteq)$. The elements of *Con*, the *constraints*, represent partial information and we shall think of them as being *assertions*. The order $\sqsubseteq$, the join $\sqcup$, the bottom *true* and the top *false* of the lattice correspond respectively to entailment, conjunction, the empty information and the join of all (possibly inconsistent) information.

Constraint systems provide the domains and operations upon which the semantic foundations of ccp calculi are built. As such, ccp operations and their logical counterparts typically have a corresponding elementary construct or operation on the elements of the constraint system. In particular, parallel composition and conjunction correspond to the *join* operation, and existential quantification and local variables correspond to a cylindrification operation on the set of constraints [39].

Similarly, the notion of computational space and the epistemic notion of belief in sccp [26] correspond to a family of functions $[\cdot]_i : Con \to Con$ on the elements of the constraint system *Con*. These functions are called *space functions*. From a computational point of view the assertion (constraint) $[c]_i$ specifies that c resides within the space of agent i. From an epistemic point of view, the assertion $[c]_i$ specifies that agent i considers c to be true (i.e. that in the world of agent i assertion c is true). Both intuitions convey the idea of c being local (subjective) to agent i.

It is therefore natural to assume that a mechanism for extrusion in ccp ought to have a corresponding semantic concept in constraint systems. Furthermore, by incorporating extrusion directly in constraint systems, the concept may become available not only to sccp but also to other declarative constraint-based formalisms.

1.2. Algebraic structures for extrusion and epistemic reasoning

Our main goal in this article is to investigate algebraic operations in spatial constraint systems that help provide semantic foundations to reason about extrusion and epistemic phenomena. From a computational point of view, the new operations will allow us to specify mobile behavior as constraints. From a logic point of view, they will allow us to specify epistemic concepts such as belief, knowledge, utterances, opinions, and intentional lies.

1.3. Contributions

In this article we generalize the underlying theory of spatial constraint systems by adding *extrusion* functions to their structure. We show that spatial constraint systems provide for the specification of spatial mobility and epistemic concepts such as belief, utterance and lies. We shall also show that the original spatial theory of sccp [26], which captures belief, can also capture an epistemic notion of knowledge. This latter contribution does not involve extrusion but it is consistent with our goal of using algebraic spatial structures to capture epistemic behavior.

Our main contributions can be summarized and structured as follows.

1. *Extrusion as the right inverse of space*. We shall first introduce a family of functions $\uparrow_i$, called *extrusion functions*. Computationally, $\uparrow_i$ can be used to intentionally extrude information from within a space $[\cdot]_i$. Epistemically, $\uparrow_i$ can be used to