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Fault-tolerant embedding of complete binary trees in locally twisted cubes☆

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HIGHLIGHTS

- We prove that the complete binary tree can be embedded with dilation 2, congestion 1, expansion 1, and load 1 into Locally twisted cube.
- We present three effective algorithms for fault-tolerant embedding of complete binary trees in locally twisted cubes with respect to one faulty node, two faulty node, and any faulty set F of $2 < |F| \leq 2^{n-1}$ nodes, respectively.

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ABSTRACT

The complete binary tree is an important network structure for parallel and distributed computing, which has many nice properties and is often used to be embedded into other interconnection architectures. The locally twisted cube LTQ_n is an important variant of the hypercube Q_n. It has many better properties than Q_n with the same number of edges and vertices. The main results obtained in this paper are: (1) The complete binary tree CBT_n rooted at an arbitrary vertex of LTQ_n can be embedded with dilation 2 and congestion 1 into LTQ_n. (2) When there exists only one faulty node in LTQ_n, both the dilation and congestion will become 2 after reconfiguring CBT_n. (3) When there exist two faulty nodes in LTQ_n, then both the dilation and congestion will become 3 after reconfiguring CBT_n. (4) For any faulty set F of LTQ_n with $2 < |F| \leq 2^{n-1}$, both the dilation and congestion will become 3 after reconfiguring CBT_n under certain constraints.

1. Introduction

Interconnection architecture is an important component in parallel computing systems, which can generally be represented by a simple graph $G = (V, E)$, where V is the vertex set and E is the edge set. In this paper, we always refer to a graph as a simple graph.

Given two graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, an embedding from G to H is an injection f from V_G to V_H. We call G the *guest graph* and H the *host graph* with respect to the embedding f. Many applications, such as architecture simulation, processor allocation, VLSI chip design can be modeled as a *graph embedding* problem [1,2,20,32,38].

The *dilation* and *expansion* are two important metrics to measure the performance of an embedding. The dilation of embedding f is defined as $dil(G, H, f) = \max\{dist(H, f(x), f(y))|(x, y) \in E_G\}$, which measures the communication delay, where $dist(H, f(x), f(y))$ denotes the distance between the two vertices $f(x)$ and $f(y)$ in H. The congestion of embedding f is defined as $C(G, H, f) = \max\{C(e)|e \in E_H\}$, which measures queuing delay of messages, where $C(e)$ denotes the number of edges in G mapped to a path in H that includes e. It is possible that the dilation and congestion of an embedding impact each other. Hence, we have to consider the tradeoff between the dilation and congestion of an embedding.

The hypercubes are one of the most popular interconnection architectures, because they have many advantageous properties such as comparably lower vertex degree and diameter, higher connec-

☆ Some preliminary results were published at MSN 2015 (Liu et al., 2015). In this version, we enhance results that fault-tolerant embedding of complete binary trees are further considered the case when the number of faulty nodes is more than two and the corresponding algorithms are presented.

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tivity and symmetry. To achieve smaller diameter with the same number of vertices and edges as the hypercubes, many variants of the hypercubes have been proposed, such as crossed cubes [7], Maöbius cubes [6], twisted cubes [15], and parity cubes [35], etc. The n-dimensional locally twisted cube (denoted by LTQ_n), proposed by Yang et al. [37], is conceptually closer to hypercube than existing variants. It has many attractive features superior to those of the n-dimensional hypercube (denoted by Q_n), such as the diameter is only about half of that of Q_n, which means the communication delay between any two vertices decrease by almost a half under the worst case. Recently, some important properties of locally twisted cubes, such as Hamiltonicity, pancyclicity, restricted connectivity, and embedding capabilities, have been deeply investigated in the literature [12,36,39,25,16,18,17].

Paths, cycles, meshes and trees are the common networks often used as guest graphs in many graph embeddings [9,24,34,8,4]. Due to the desirable performance and wide applications of the complete binary tree, its embeddability into various interconnection architectures catches even more attention. So far, much work about the embeddings of the complete binary tree into meshes, star graphs, lines, grids, butterfly networks and hypercubes has been explored in the literature [10,33,23,27,11,14]. However, embeddings of the complete binary tree into a few of all the existing hypercube variants have been studied. It has been proven that the complete binary tree can be embedded with dilation 1 and expansion 1 into folded cubes, enhanced cubes, crossed cubes, parity cubes, and Möbius cubes, respectively [5,22,28,29]. It has been proven that the complete binary tree cannot be embedded into a hypercube with dilation 1 and expansion 1, but it can be embedded into a hypercube with either expansion 2 and dilation 1 or with expansion 1 and dilation 2 [31].

As the size of interconnection networks increases continuously, dealing with networks with faulty elements becomes unavoidable. In order to maintain the reliability of networks, whenever a node is found to be faulty, it should be replaced by a fault-free node. Therefore, it is important to reconfigure a guest graph in a faulty host graph where all faulty nodes have been replaced. It means to find a fault-tolerant embedding of a guest graph into host graph. Much work about fault-tolerant embeddings of the complete binary tree into meshes, hypercubes, star graphs, complete transposition graphs and bubblesort graphs has been explored in the literature [10,3,26,19].

In this paper, we study the embedding of complete binary trees into locally twisted cubes. Firstly, we prove that the complete binary trees rooted at an arbitrary vertex of LTQ_n can be embedded with dilation 2 and congestion 1 into LTQ_n. Furthermore, we present three effective algorithms for fault-tolerant embedding of complete binary trees in locally twisted cubes. We prove that both the dilation and congestion will become 2 after reconfiguring CBT_n when there exists only one faulty node in LTQ_n, while both the dilation and congestion will become 3 after reconfiguring CBT_n when there exist two faulty nodes in LTQ_n. For any faulty set F of LTQ_n with $2 < |F| \leq 2^{n-1}$, both the dilation and congestion will become 3 after reconfiguring CBT_n under certain constraints.

The rest of this paper is organized as follows: Section 2 provides the preliminaries. Section 3 proves that the complete binary tree CBT_n can be embedded with dilation 2 and congestion 1 into n-dimensional locally twisted cube. Section 4 proves that complete binary trees can be reconfigured dynamically in locally twisted cubes with low cost. Section 5 presents three effective algorithms for fault-tolerant embedding of complete binary trees in locally twisted cubes. The final section concludes this paper.

2. Preliminaries

In this section, we will give some terminologies, definitions and basic lemmas used in this paper. Given two simple graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, H is said to be a *subgraph* of G if $V_H \subseteq V_G$ and $E_H \subseteq E_G$. The subgraph *induced* by V' in G is denoted by $G[V']$, where $V' \subseteq V_G$.

For any integer $n \geq 1$, a binary string x of length n is denoted by $x_{n-1} \ldots x_i \ldots x_1 x_0 (0 \leq i \leq n-1)$, where $x_i \in \{0, 1\}$ is said to be the ith bit of x and $x_{n-1} \ldots x_k (1 \leq k \leq n-1)$ is called a prefix of x, and furthermore, x can be written as $(x_{n-1} \ldots x_k) x_{k-1} \ldots x_1 x_0$. The ith bit of x can also be denoted as $bit(x, i)$. The *complement* of x_i is denoted by $\overline{x_i} = 1 - x_i$. The length of x is denoted by $|x|$.

The n-dimensional locally twisted cube, denoted by LTQ_n, is an n-regular graph of 2^n vertices and 2^n edges. Every vertex of LTQ_n is identified by a unique binary string of length n. We adopt the recursive definition of locally twisted cube from [37].

Definition 1 ([37]). Let $n \geq 2$. The n-dimensional locally twisted cube, LTQ_n, is defined recursively as follows.

(1) LTQ_2 is a graph consisting of four vertices labeled with 00, 01, 10, and 11, respectively, connected by four edges (00, 01), (00, 10), (01, 11), and (10, 11);
(2) For $n \geq 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps. Let LTQ^0_{n-1} denote the graph obtained by prefixing the label of each vertex of one copy of LTQ_{n-1} with 0, let LTQ^1_{n-1} denote the graph obtained by prefixing the label of each vertex of the other copy of LTQ_{n-1} with 1. Connect each vertex $x = 0x_{n-2}x_{n-3} \ldots x_0$ of LTQ^0_{n-1} with the vertex $1(x_{n-2} + x_0)x_{n-3} \ldots x_0$ of LTQ^1_{n-1} with an edge, where '+' represents the modulo 2 addition.

Fig. 1(a) and (b) illustrate LTQ_3 and LTQ_4, respectively.

By Definition 1, we can easily check if a given pair of vertices are adjacent in LTQ_n. When two adjacent vertices u and v have a leftmost differing bit at position i, we say that v is the i-neighbor of u, denoted by $N_i(u)$, the edge (u, v) is an i-dimensional edge, or u and v are adjacent along dimension i. For example, letting $u = 0101$, then $N_0(u) = 0100$, $N_3(u) = 1001$.

Yang et al. have found an isomorphic expression of LTQ_n. For example, two graphs shown in Fig. 2(a) and (b) are other expressions of LTQ_3 and LTQ_4, respectively. In general, they have proven the following result.

Lemma 2 ([37]). *Let Q_l be the graph obtained from Q_{n-1} by suffixing the labels of all vertices with 0, Q_r be the graph obtained from a graph isomorphic to Q_{n-1} by suffixing the labels of all vertices with 1. Then LTQ_n is isomorphic to the graph obtained from Q_l and Q_r by adding a perfect matching M between them, denoted by $LTQ_n = Q_l \oplus Q_r$, where M is the set of edges by linking two vertices with difference only suffixes.*

The complete binary tree has many nice properties than other binary trees. The perfect binary tree may be seen as a complete binary tree with $2^n - 1$ nodes, where n is its height. It is regular and each internal vertex has exactly two children, and all leaves have the same level. It is more important that each binary tree is a subtree of the perfect binary tree with the same height. In this paper, we use perfect binary trees and complete binary trees interchangeably.

The double-rooted complete binary tree of height n (or briefly a DT_n) is a graph consisting of two complete binary trees of height $n-1$ whose roots are connected by a path of length 3. Clearly, a double-rooted complete binary tree of height n has exactly 2^n vertices. Note that if we identify both roots of a double-rooted complete binary tree, we obtain a complete binary tree of the same height.