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On g-good-neighbor conditional diagnosability of (n, k)-star networks ☆

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ABSTRACT

The g-good-neighbor conditional diagnosability is a new measure for fault diagnosis of systems. Xu et al. (2017) [27] determined the g-good-neighbor conditional diagnosability of (n, k)-star networks $S_{n,k}$ (i.e., $t_g(S_{n,k})$) with $1 \le k \le n-1$ for $1 \le g \le n-k$ under the PMC model and the MM* model. In this paper, we determine $t_g(S_{n,k})$ for all the remaining cases with $1 \le k \le n-1$ for $1 \le g \le n-1$ under the two models, from which we can obtain the g-good-neighbor conditional diagnosability of the star graph obtained by Li et al. (2017) [16] for $1 \le g \le n-2$.

1. Introduction

With the size of multiprocessor systems increasing, processor failure is inevitable. Thus, to evaluate the reliability of multiprocessor systems, fault diagnosability has become an important metric. Many models have been proposed for determining a multiprocessor system's diagnosability. The PMC model was proposed by Preparata, Metze, and Chien [23] for fault diagnosis in multiprocessor systems. In the PMC model, all processors in the system under diagnosis can test one another. The MM model, proposed by Maeng and Malek [21], assumes that a vertex in the system sends the same task to two of its neighbors and then compares their responses. Sengupta and Dahbura [24] further suggested a modification of the MM model, called the MM* model, in which each processor has to test two processors if the processor is adjacent to the latter two processors. Many researchers have applied the PMC model and the MM* model to identify faults in various topologies (see [4–8,11–14,16,17,20,22,25–35]).

The classical diagnosability for multiprocessor systems assumes that all the neighbors of any processor may fail simultaneously. However, the probability that this event occurs is very small in large-scale multiprocessor systems. Consequently, the classical diagnosability for multiprocessor systems is upper bounded by its minimum degree and underestimates the resilience of large networks. To overcome the shortcoming, in 2005, Lai et al. [17] introduced conditional diagnosability under the assumption that all the neighbors of any processor in a multiprocessor system cannot be faulty at the same time and determined the conditional diagnosability of the n-dimensional hypercube Q_n under the PMC model is $4n-7$ for $n \ge 5$. The conditional diagnosability of interconnection networks has been widely investigated (see [4,6–8,13,28–30,33–35]).

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Table 1
The g-good-neighbor conditional diagnosability of $S_{n,k}$ under the PMC model and the MM* model.

	$k=1$	$k=2$	$3\le k\le n-2$	$k=n-1$
$g=1$	$\left\lceil \frac{n}{2}\right\rceil-1$ $(n\ge 4)$ [27] ? $(n=3)$	n (PMC) [27] ? $(n\ge 3$, MM*$)$	$n+k-2$ [27]	$2n-3$ [16,27]
$2\le g\le \left\lfloor \frac{n}{2}\right\rfloor-1$	$\left\lceil \frac{n}{2}\right\rceil-1$ [27]	$n+g(k-1)-1$ [27]		$(n-g)(g+1)!-1$ [16]
$\left\lfloor \frac{n}{2}\right\rfloor\le g\le n-k$	?			
$n-k\le g\le n-2$	Nonexistence	?		
$g=n-1$	0			

Table 2
The g-good-neighbor conditional diagnosability of $S_{n,k}$ under the PMC model and the MM* model.

	$k=1$	$k=2$	$3\le k\le n-2$	$k=n-1$
$g=1$	$\left\lceil \frac{n}{2}\right\rceil-1$ $(n\ge 4)$ [27] 0 $(n=3$, MM*$)$ (Theorem 3.6) 1 $(n=3$, PMC$)$ (Theorem 3.6)	n (PMC) [27] $n-1$ $(n\ge 4$, MM*$)$ (Theorem 3.7) 1 $(n=3$, MM*$)$ (Theorem 3.7)	$n+k-2$ [27]	$2n-3$ [16,27]
$2\le g\le \left\lfloor \frac{n}{2}\right\rfloor-1$	$\left\lceil \frac{n}{2}\right\rceil-1$ [27]	$n+g(k-1)-1$ [27]		$(n-g)(g+1)!-1$ [16]
$\left\lfloor \frac{n}{2}\right\rfloor\le g\le n-k$	$n-g-1$ (Theorem 3.6)			
$n-k\le g\le n-2$	Nonexistence	$\frac{(g+1)!(n-g)}{(n-k)!}-1$ $(n\ge 4)$ (Theorem 3.5)		
$g=n-1$	0			

In 2012, Peng et al. proposed g-good-neighbor conditional diagnosability [22], which extended the concept of conditional diagnosability. This requires that every fault-free vertex has at least g fault-free neighbors. Peng et al. [22] showed the g-good-neighbor conditional diagnosability of the n-dimensional hypercube Q_n under the PMC model is $2^g(n-g+1)-1$, which is several times larger than the classical diagnosability of Q_n depending on the condition g. Compared with the classical diagnosability and the conditional diagnosability of interconnection networks, the g-good-neighbor conditional diagnosability can provide more accurate measurement of diagnosability for a large-scale processing system. Since then, many researchers have studied this topic (see [16,20,25–27,31,32]).

The (n,k)-star network $S_{n,k}$, proposed by Chiang and Chen [3], is an extension of the n-dimensional star graph S_n. The network $S_{n,k}$ preserves many ideal properties of S_n. In recent years, $S_{n,k}$ has received considerable attention [4,7–9,15,18,19,27,33]. In particular, Xu et al. [27] derived the following result about the g-good-neighbor conditional diagnosability of $S_{n,k}$ under the PMC model and the MM* model.

Theorem 1.1 *(Xu et al. [27]). The g-good-neighbor conditional diagnosabilities of the (n,k)-star graph $S_{n,k}$ under the PMC model and the MM* model are*

$$t_g(S_{n,k})=\begin{cases}\left\lceil \frac{n}{2}\right\rceil-1 & \text{if } 1\le g\le \left\lfloor \frac{n}{2}\right\rfloor-1,\ k=1,\ n\ge 4;\\ n+g(k-1)-1 & \text{if } 1\le g\le n-k,\ 2\le k\le n-1,\end{cases}$$

and

$$t_g(S_{n,k})=\begin{cases}\left\lceil \frac{n}{2}\right\rceil-1 & \text{if } 1\le g\le \left\lfloor \frac{n}{2}\right\rfloor-1,\ k=1,\ n\ge 4;\\ n+k-2 & \text{if } g=1,\ 3\le k\le n-1,\ n\ge 4;\\ n+g(k-1)-1 & \text{if } 2\le g\le n-k,\ 2\le k\le n-1,\end{cases}$$

respectively.

However, there are some unknown cases (see Table 1).

In order to give a complete result on the g-good-neighbor conditional diagnosability of (n,k)-star networks $S_{n,k}$, we determine all the remaining cases (see Table 2). Recently, Li et al. [16] determined the g-good-neighbor conditional diagnosability of the star graph S_n under the PMC model and the MM* model as follows.

Theorem 1.2 *(Li et al. [16]). The g-good-neighbor conditional diagnosabilities of the star graph S_n with $n\ge 4$ for $0\le g\le n-2$ under the PMC model and the MM* model are $(n-g)(g+1)!-1$.*