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The g-good-neighbor diagnosability of locally twisted cubes ☆

Yunxia Ren, Shiying Wang *

Engineering Laboratory for Big Data Statistical Analysis and Optimal Control, School of Mathematics and Information Science, Henan Normal University, Xinxiang, Henan 453007, PR China

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ABSTRACT

Diagnosability of a multiprocessor system is one important measure of the reliability of interconnection networks. In 2012, Peng et al. proposed the g-good-neighbor diagnosability that restrains every fault-free node containing at least g fault-free neighbors. The locally twisted cube LTQ_n is applied widely. In this paper, we give that the g-good-neighbor diagnosability of LTQ_n is $2^g(n-g+1)-1$ under the PMC model and the MM* model for $n \geq 3$ and $0 \leq g \leq n-3$.

1. Introduction

The Internet has become an indispensable part of our lives. Processors or communication links failure of a multiprocessor system give our life a lot of trouble. How to find out the fault points accurately and timely becomes the primary problem to solve the stability of network. Therefore, the system diagnosis arises at the historic moment. The diagnosis of the system is the process of identifying the faulty processors from the fault-free ones. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of a system G is the maximum value of t such that G is t-diagnosable [2–4,7]. There are two well-known diagnosis models, one is the PMC diagnosis model, introduced by Preparata et al. [12] and the other is the MM model, proposed by Maeng and Malek [10]. In the PMC model, any two processors can test each other. To diagnose a system, we can compare their responses after a node sends the same task to two of its neighbors in the MM model. Sengupta and Dahbura [14] suggested a further modification of the MM model, called the MM* model, in which each node must test another two neighbors. The researchers have done a lot of research under the PMC model and the MM* model. In 2005, Lai et al. [7] introduced a restricted diagnosability of multiprocessor systems called the conditional diagnosability. They considered the situation that any fault set cannot contain all the neighbors of any vertex in a system. In 2012, Peng et al. [11] proposed a new measure for fault diagnosis of systems, namely, the g-good-neighbor diagnosability (which is also called the g-good-neighbor conditional diagnosability), which requires that every fault-free node contains at least g fault-free neighbors. In [11], they studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the PMC model. In 2016, Wang and Han [18] studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the MM* model. In 2016, Xu et al. [22] studied the g-good-neighbor diagnosability of complete cubic networks under the PMC model and the MM* model. Liu et al. [9] studied that the g-good-neighbor diagnosability of the exchanged hypercube under the PMC model

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* Corresponding author.
E-mail addresses: wangshiying@htu.cn, shiying@sxu.edu.cn (S. Wang).

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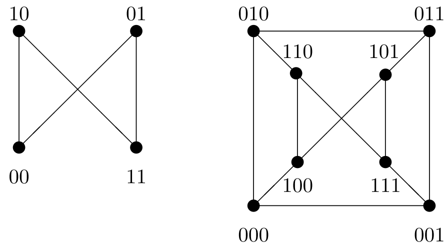

Fig. 1. LTQ_2 and LTQ_3.

and the MM* model. In 2017, Wang et al. [19,20] studied the 2-good-neighbor diagnosability of bubble-sort star graph networks under the PMC model and the MM* model and the 2-good-neighbor (2-extra) diagnosability of alternating group graph networks under the PMC model and the MM* model. Yuan et al. [24,25] studied the g-good-neighbor diagnosability of the k-ary n-cube ($k \geq 3$) under the PMC model and the MM* model. In [15,16], Wang et al. studied the g-good-neighbor diagnosability of Cayley graph generated by the transposition tree under the PMC model and the MM* model for $g = 1, 2$. In [17], Wang et al. studied the 1-good-neighbor connectivity and diagnosability of Cayley graphs generated by complete graphs. In this paper, the g-good-neighbor diagnosability of the locally twisted cube LTQ_n under the PMC model and the MM* model has been studied. It is proved that the g-good-neighbor diagnosability of LTQ_n is $2^g(n-g+1)-1$ under the PMC model and the MM* model for $n \geq 3$ and $0 \leq g \leq n-3$.

2. Preliminaries

2.1. Notations

In this paper, a multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Suppose that V' is a nonempty vertex subset of V. The induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and whose edge set consists of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v in G is the number of edges incident with v. We denote by $\delta(G)$ the minimum degree of vertices of G. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor vertex or a neighbor of v for $u \in N_G(v)$. Let $S \subseteq V(G)$. We denote by $N_G(S)$ the set $\cup_{v\in S} N_G(v) \setminus S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if $d_G(v) = k$ for any vertex $v \in V$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. Let $G = (V, E)$. A fault set $F \subseteq V$ is called a g-good-neighbor conditional faulty set if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A g-good-neighbor cut of a graph G is a g-good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is said to be the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. Let F_1 and F_2 be two distinct subsets of V, and let the symmetric difference $F_1 \triangle F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. For graph-theoretical terminology and notation not defined here we follow [1].

2.2. Locally twisted cubes

For an integer $n \geq 1$, a binary string of length n is denoted by $u_1u_2 \ldots u_n$, where $u_i \in \{0, 1\}$ for any integer $i \in \{1, 2, \ldots, n\}$. The n-dimensional locally twisted cube, denoted by LTQ_n, is an n-regular graph of 2^n vertices and $n2^{n-1}$ edges, which can be recursively defined as follows [23].

Definition 2.1. ([23]) For $n \geq 2$, an n-dimensional locally twisted cube, denoted by LTQ_n, is defined recursively as follows:

1) LTQ_2 is a graph consisting of four nodes labeled with 00, 01, 10 and 11, respectively, connected by four edges {00, 01}, {01, 11}, {11, 10} and {10, 00}.

2) For $n \geq 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps: Let $0LTQ_{n-1}$ denote the graph obtained from one copy of LTQ_{n-1} by prefixing the label of each node with 0. Let $1LTQ_{n-1}$ denote the graph obtained from the other copy of LTQ_{n-1} by prefixing the label of each node with 1. Connect each node $0u_2u_3 \cdots u_n$ of $0LTQ_{n-1}$ to the node $1(u_2 + u_n)u_3 \cdots u_n$ of $1LTQ_{n-1}$ with an edge, where "+" represents the modulo 2 addition.

The edges whose end vertices in different $iLTQ_{n-1}s$ are the cross-edges.

Figs. 1 and 2 show three examples of locally twisted cubes. The locally twisted cube can also be equivalently defined in the following non-recursive fashion.

Definition 2.2. ([23]) For $n \geq 2$, the n-dimensional locally twisted cube, denoted by LTQ_n, is a graph with $\{0, 1\}^n$ as the node set. Two nodes $u_1u_2 \cdots u_n$ and $v_1v_2 \cdots v_n$ of LTQ_n are adjacent if and only if either one of the following conditions is satisfied.