Accepted Manuscript

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PII: S0304-3975(17)30524-8
DOI: http://dx.doi.org/10.1016/j.tcs.2017.06.012
Reference: TCS 11226

To appear in: *Theoretical Computer Science*

Received date: 22 February 2017
Revised date: 22 May 2017
Accepted date: 19 June 2017

Please cite this article in press as: A. Munaro, Boundary classes for graph problems involving non-local properties, *Theoret. Comput. Sci.* (2017), http://dx.doi.org/10.1016/j.tcs.2017.06.012

Boundary Classes for Graph Problems Involving Non-Local Properties

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Abstract

We continue the study of boundary classes for NP-hard problems and focus on seven NP-hard graph problems involving non-local properties: Hamiltonian Cycle, Hamiltonian Cycle Through Specified Edge, Hamiltonian Path, Feedback Vertex Set, Connected Vertex Cover, Connected Dominating Set and Graph VC_{con} Dimension. Our main result is the determination of the first boundary class for Feedback Vertex Set. We also determine boundary classes for Hamiltonian Cycle Through Specified Edge and Hamiltonian Path and give some insights on the structure of some boundary classes for the remaining problems.

Keywords: Computational complexity, Hereditary class, Boundary class, Hamiltonian Cycle, Feedback Vertex Set

1. Introduction

Many NP-hard graph problems remain NP-hard even for restricted classes of graphs, while they become polynomial-time solvable when further restrictions are applied. For example, the well-known graph problem Hamiltonian Cycle is NP-hard in general and it remains NP-hard for subcubic graphs (see, e.g., [26]). On the other hand, it is clearly trivial for graphs with maximum degree 2. It is therefore natural to ask when a certain "hard" graph problem becomes "easy": Is there any "boundary" separating "easy" and "hard" instances? Alekseev [3] considered this question in the case the instances are hereditary classes of graphs. Given a graph problem Π, a hereditary class of graphs X is Π-*hard* if Π is NP-hard for X, and Π-*easy* if Π is solvable in polynomial time for graphs in X. He introduced the notion of Π-boundary class, playing the role of the "boundary" separating Π-hard and Π-easy instances, and showed that a finitely defined (hereditary) class is Π-hard if and only if it contains a Π-boundary class. Moreover, he determined a boundary class for Independent Set, the first result in the systematic study of boundary classes for NP-hard graph problems (see, e.g., [6, 7, 38, 50, 51]). Note that here and throughout the paper we tacitly assume that $\mathsf{P} \neq \mathsf{NP}$, or else the notion of "boundary" becomes vacuous.

In this paper, we continue the study of boundary classes for NP-hard problems and focus on seven NP-hard graph problems involving non-local properties: Hamiltonian Cycle, Hamiltonian Cycle Through Specified Edge, Hamiltonian Path, Feedback Vertex Set, Connected Vertex Cover, Connected Dominating Set and Graph VC_{con} Dimension. Our main result is the determination of the first boundary class for Feedback Vertex Set.

In a first attempt to answer the meta-question posed above, one might be tempted to consider maximal Π-easy classes and minimal Π-hard classes. In fact, the first approach immediately turns out to be meaningless: there are no maximal Π-easy classes. Indeed, every Π-easy class X can be extended to another Π-easy class simply by adding to X a graph $G \notin X$ together with all its induced subgraphs. Even the approach through minimal Π-hard classes is not completely satisfactory, as for some problems they might not exist at all:

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