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Boundary classes for graph problems involving non-local properties

Andrea Munaro

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# 1 Boundary Classes for Graph Problems Involving Non-Local Properties

2 Andrea Munaro

3 *Laboratoire G-SCOP, Univ. Grenoble Alpes*

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## 4 Abstract

We continue the study of boundary classes for NP-hard problems and focus on seven NP-hard graph problems involving non-local properties: HAMILTONIAN CYCLE, HAMILTONIAN CYCLE THROUGH SPECIFIED EDGE, HAMILTONIAN PATH, FEEDBACK VERTEX SET, CONNECTED VERTEX COVER, CONNECTED DOMINATING SET and GRAPH  $VC_{\text{con}}$  DIMENSION. Our main result is the determination of the first boundary class for FEEDBACK VERTEX SET. We also determine boundary classes for HAMILTONIAN CYCLE THROUGH SPECIFIED EDGE and HAMILTONIAN PATH and give some insights on the structure of some boundary classes for the remaining problems.

5 *Keywords:* Computational complexity, Hereditary class, Boundary class, Hamiltonian Cycle, Feedback  
6 Vertex Set

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## 7 1. Introduction

8 Many NP-hard graph problems remain NP-hard even for restricted classes of graphs, while they become  
9 polynomial-time solvable when further restrictions are applied. For example, the well-known graph problem  
10 HAMILTONIAN CYCLE is NP-hard in general and it remains NP-hard for subcubic graphs (see, e.g., [26]). On  
11 the other hand, it is clearly trivial for graphs with maximum degree 2. It is therefore natural to ask when  
12 a certain “hard” graph problem becomes “easy”: Is there any “boundary” separating “easy” and “hard”  
13 instances? Alekseev [3] considered this question in the case the instances are hereditary classes of graphs.  
14 Given a graph problem  $\Pi$ , a hereditary class of graphs  $X$  is  $\Pi$ -hard if  $\Pi$  is NP-hard for  $X$ , and  $\Pi$ -easy if  $\Pi$  is  
15 solvable in polynomial time for graphs in  $X$ . He introduced the notion of  $\Pi$ -boundary class, playing the role  
16 of the “boundary” separating  $\Pi$ -hard and  $\Pi$ -easy instances, and showed that a finitely defined (hereditary)  
17 class is  $\Pi$ -hard if and only if it contains a  $\Pi$ -boundary class. Moreover, he determined a boundary class for  
18 INDEPENDENT SET, the first result in the systematic study of boundary classes for NP-hard graph problems  
19 (see, e.g., [6, 7, 38, 50, 51]). Note that here and throughout the paper we tacitly assume that  $P \neq NP$ , or  
20 else the notion of “boundary” becomes vacuous.

21 In this paper, we continue the study of boundary classes for NP-hard problems and focus on seven NP-hard  
22 graph problems involving non-local properties: HAMILTONIAN CYCLE, HAMILTONIAN CYCLE THROUGH  
23 SPECIFIED EDGE, HAMILTONIAN PATH, FEEDBACK VERTEX SET, CONNECTED VERTEX COVER, CON-  
24 NECTED DOMINATING SET and GRAPH  $VC_{\text{con}}$  DIMENSION. Our main result is the determination of the  
25 first boundary class for FEEDBACK VERTEX SET.

26 In a first attempt to answer the meta-question posed above, one might be tempted to consider maximal  $\Pi$ -  
27 easy classes and minimal  $\Pi$ -hard classes. In fact, the first approach immediately turns out to be meaningless:  
28 there are no maximal  $\Pi$ -easy classes. Indeed, every  $\Pi$ -easy class  $X$  can be extended to another  $\Pi$ -easy class  
29 simply by adding to  $X$  a graph  $G \notin X$  together with all its induced subgraphs. Even the approach through  
30 minimal  $\Pi$ -hard classes is not completely satisfactory, as for some problems they might not exist at all:

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*Email address:* aerdna.munaro@gmail.com (Andrea Munaro)

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